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NASA TECHNICAL MEMORANDUM

REPORT NO. 53904

IMPULSIVE VELOCITY REQUIREMENTS FOR INSERTION INTO ORBITS ABOUT VENUS, MARS, AND JUPITER

By Richard Gold
Aero-Astrodynamics Laboratory

September 12, 1969

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

MSFC - Form 3190 (September 1968)

IN-AERO-68-3

was changed to TM X-53904 on September 12, 1969

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ABSTRACT

The tangential impulsive coplanar transfer between a planetary approach hyperbolic trajectory and a planetocentric capture orbit of a desired size and orientation is studied for Venus, Mars, and Jupiter. The parameters considered are rotation angle, ROT, periapsis radius, R_p, apsidal ratio of the capture orbit, N, and hyperbolic approach velocity, $\bar{V}_\infty$. This study is intended to serve as an aid to mission analysts and spacecraft designers in the area of preliminary design. The method used in computing the impulsive velocity requirements is presented in the appendix.

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IN-AERO-68-3

July 19, 1968

IMPULSIVE VELOCITY REQUIREMENTS FOR INSERTION INTO ORBITS ABOUT VENUS, MARS, AND JUPITER

By

Richard Gold

ASTRODYNAMICS AND MISSION ANALYSIS GROUP
ADVANCED STUDIES OFFICE
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Cont'd)

NOTE: The figures for Venus and Mars have R_p of 1.1, 2 and 5 planetary radii and apsidal ratios of 2, 4, 6, 8, and 10. The figures for Jupiter have R_p of 1.1, 1.3, 3, 5, 10, and 20 Jovian radii and apsidal ratios of 3, 5, 10, and 20. The figures are arranged so that all the apsidal ratios are presented for one R_p before the next R_p is considered.

For example: Figure 3 shows data for an R_p of 1.1 Venus radii and an apsidal ratio of 2. Figure 4 shows data for an R_p of 1.1 Venus radii and an apsidal ratio of 4, etc.

DEFINITION OF SYMBOLS

Symbol	Definition
ROT	angle between the major axis of the approach hyperbola and the line of apsides of a capture orbit
$\bar{R}_{ph}$	periapsis radius of the incoming hyperbola
$\bar{R}_p$	periapsis radius of the capture orbit
$\bar{R}_a$	apoapsis radius of the capture orbit
N	apsidal ratio of the capture orbit (R_a/R_p)
$\bar{V}\infty$	hyperbolic approach velocity
$\hat{S}$in, $\hat{S}$out	the incoming and outgoing hyperbolic asymptote direction
θ	angle between the major axis of the approach hyperbola and the outgoing asymptote
ψ	angle between the line of apsides of the capture orbit and the incoming asymptote
$\bar{V}_h$	velocity on the hyperbola at point of insertion
$\bar{V}_e$	velocity on the capture orbit at point of insertion
$\overline{\Delta V}$	the vectorial difference between the $\bar{V}_h$ and $\bar{V}_e$

IMPULSIVE VELOCITY REQUIREMENTS FOR INSERTION INTO ORBITS ABOUT VENUS, MARS, AND JUPITER

SUMMARY

The tangential impulsive coplanar transfer between a planetary approach hyperbolic trajectory and a planetocentric capture orbit of a desired size and orientation is studied for Venus, Mars, and Jupiter. The parameters considered are rotation angle, ROT, periapsis radius, R_p, apsidal ratio of the capture orbit, N, and hyperbolic approach velocity, $\bar{V}_\infty$. Results, which are intended to serve as an aid to mission analysts and spacecraft designers in the area of preliminary design, are presented graphically for Venus, Mars, and Jupiter. The method used in computing the impulsive velocity requirements is presented in the appendix.

INTRODUCTION

For an interplanetary mission, once the launch and arrival dates are chosen, the magnitude of the periapsis velocity and area of periapsis passage of the approach hyperbola are determined. If the desired location of the periapsis of the capture orbit lies outside the area of periapsis passage of the approach hyperbola, a rotation of the line of apsides of the capture orbit is necessary. The line of apsides can be rotated either after braking into some intermediate capture orbit, or at the time of orbit insertion. The latter method yields the least total impulsive velocity requirements.

The transfer between an incoming hyperbola and a capture orbit can occur only when the two orbits intersect. When the two orbits are coplanar, they can intersect in either one or two places. Figure 1 depicts the situation where the periapsis of the approach hyperbola falls below the local radius of the capture orbit, resulting in two points of intersection, A and B. Figure 2 shows the case where the approach hyperbola and capture orbit touch at only one point. This is called the tangential case. A detailed development of the mathematical model for both cases is given in the appendix.

DISCUSSION

Results have been obtained for a range of hyperbolic approach velocities, apsidal ratios, and capture orbit periapsides. To be of aid to mission analysts and spacecraft designers in the area of preliminary design, the results are presented in graphical form.

Data were generated for coplanar tangential insertion into orbits about Venus, Mars, and Jupiter. The data are presented for each planet in three forms. First, the relation between rotation angle and tangential insertion velocity requirements is shown for a range of fixed approach hyperbolic velocities. Second, the relation between approach hyperbolic velocity, and tangential insertion velocity requirements is presented for several fixed rotation angles. Third, the effect of rotation angle on the periapsis radius of the approach hyperbola is presented for a range of apsidal ratios. Because the periapsis radius of the approach hyperbola is relatively insensitive to the hyperbolic excess velocity, only one hyperbolic excess velocity is considered for the third set of data.

To find a minimum hyperbolic approach velocity, Hohmann transfers were calculated from earth to each of the planets considered. In this calculation, the planets were considered to be in circular coplanar orbits about the sun with orbital radii equal to the semi-major axis of the elliptical orbits of the planets. For Venus the approach hyperbolic velocity for the Hohmann case is 2.76 km/sec, for Mars, 2.65 km/sec, and for Jupiter, 5.64 km/sec. A slightly lower initial hyperbolic approach velocity value was used in this parametric study.

The parameters of the capture orbit (capture orbit periapsis radius, and apsidal ratio), and of the approach hyperbola (hyperbolic approach velocity) were varied for a range of rotation angles for each planet. For Venus and Mars, the capture orbit periapsis radii considered were 1.1, 2, and 5 planetary radii, and the apsidal ratios considered were 2, 4, 6, 8, and 10. For Jupiter the capture orbit periapsis radii considered were 1.1, 1.3, 3, 5, 10, and 20 Jovian radii and the apsidal ratios considered were 3, 5, 10, and 20.

USE OF DATA

These data can be used in orbit selection for a spacecraft, spacecraft sizing for new interplanetary missions, and evaluation of proposed interplanetary missions.

If a spacecraft which has an impulsive velocity capability of 4 km/sec should already exist, a Mars mission can easily be evaluated. Consider a candidate mission to a Mars orbit with a hyperbolic approach velocity of 6 km/sec and a Mars capture orbit with a periapsis radius of 1.1 Mars radii and an apsidal ratio of 8. Can the spacecraft brake into orbit if, for instance, a 37.5 degree rotation of the line of apsides of the capture orbit is necessary? From figure 39, the velocity requirement, which is 3.4 km/sec, can be easily read; therefore, the spacecraft can insert into orbit. The radius of transfer, from figure 66, is 1.4 Mars radii.

If a desired mission, including depart and arrival times (thus fixing the arrival hyperbolic approach velocity) and a desired capture orbit are chosen, a spacecraft can be designed to allow for a desired rotation of the line of apsides. As another example, let us suppose a Venus mission is chosen where the approach hyperbolic velocity is 4 km/sec, and where the desired parking orbit has a periapsis radius of 1.1 Venus radii and an apsidal ratio of 2. If the desired rotation of the line of apsides is 60 degrees, then what is the magnitude of the required braking velocity impulse? From figure 3, the required braking velocity impulse is 3.6 km/sec. Figure 33 shows that the radius of transfer is 1.36 Venus radii.

If a mission is proposed where the required rotation angle and capture orbit parameters are constant but the arrival hyperbolic velocity fluctuates, the second type of graph will show the necessary insertion velocity. For instance, if a capture orbit has a periapsis radius of 1.1 Venus radii, an apsidal ratio of 2, a desired rotation of the line of apsides of 60 degrees, and the maximum hyperbolic approach velocity is 5.5 km/sec, what is the necessary insertion velocity? From figure 18, the required velocity increment is 3.76 km/sec.

CONCLUSION

There are several ways to rotate the line of apsides of a capture orbit away from the major axis of an approach hyperbola. In the method considered here, the rotation occurs at the time of orbit insertion. A range of incoming hyperbolic and capture orbit parameters is considered. The tangential impulsive coplanar transfer is within 3 percent of the optimum insertion ΔV for small rotation angles (see reference 1).

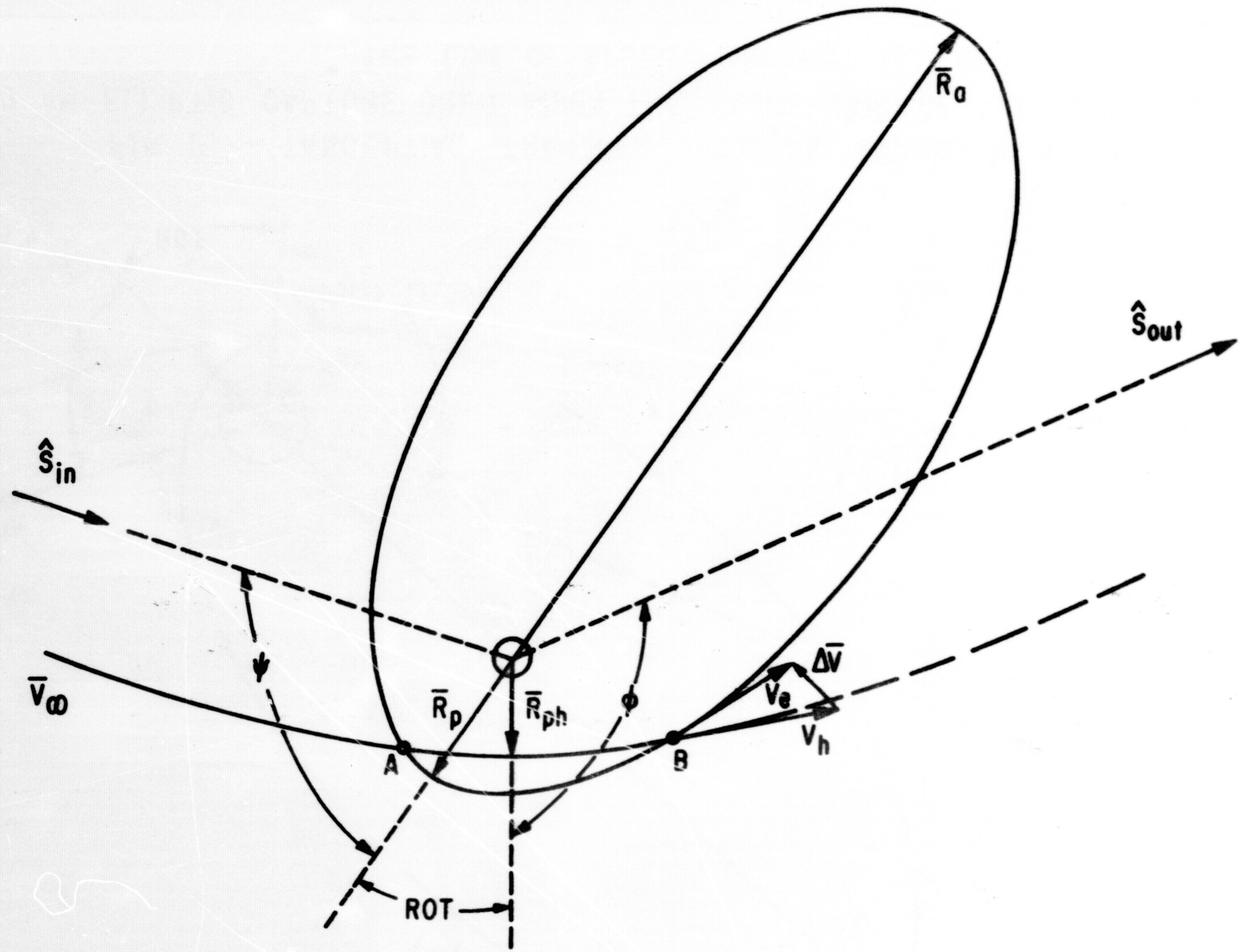


FIG. 1. TRANSFER FROM AN APPROACH HYPERBOLA TO A CAPTURE ORBIT WHERE BOTH THE MAJOR AXIS OF THE HYPERBOLA AND THE LINE OF APSIDES ARE NOT ALIGNED AND WHERE THE $\bar{R}_{ph}$ IS CONSTRAINED

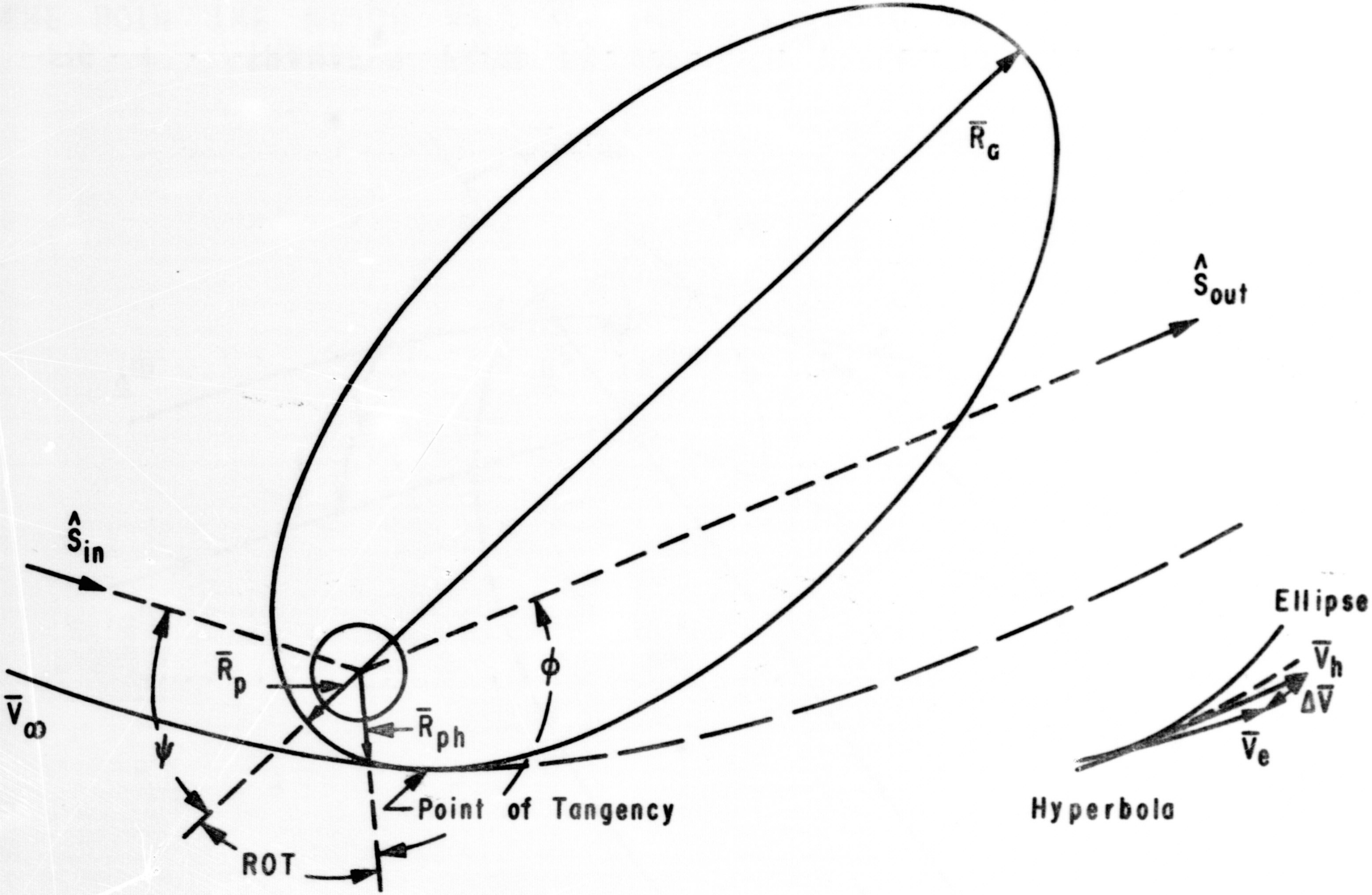


FIG. 2. TANGENTIAL TRANSFER FROM AN APPROACH HYPERBOLA TO AN ELLIPTIC CAPTURE ORBIT WHEN THE MAJOR AXIS OF THE HYPERBOLA AND THE LINE OF APSIDES ARE NOT ALIGNED

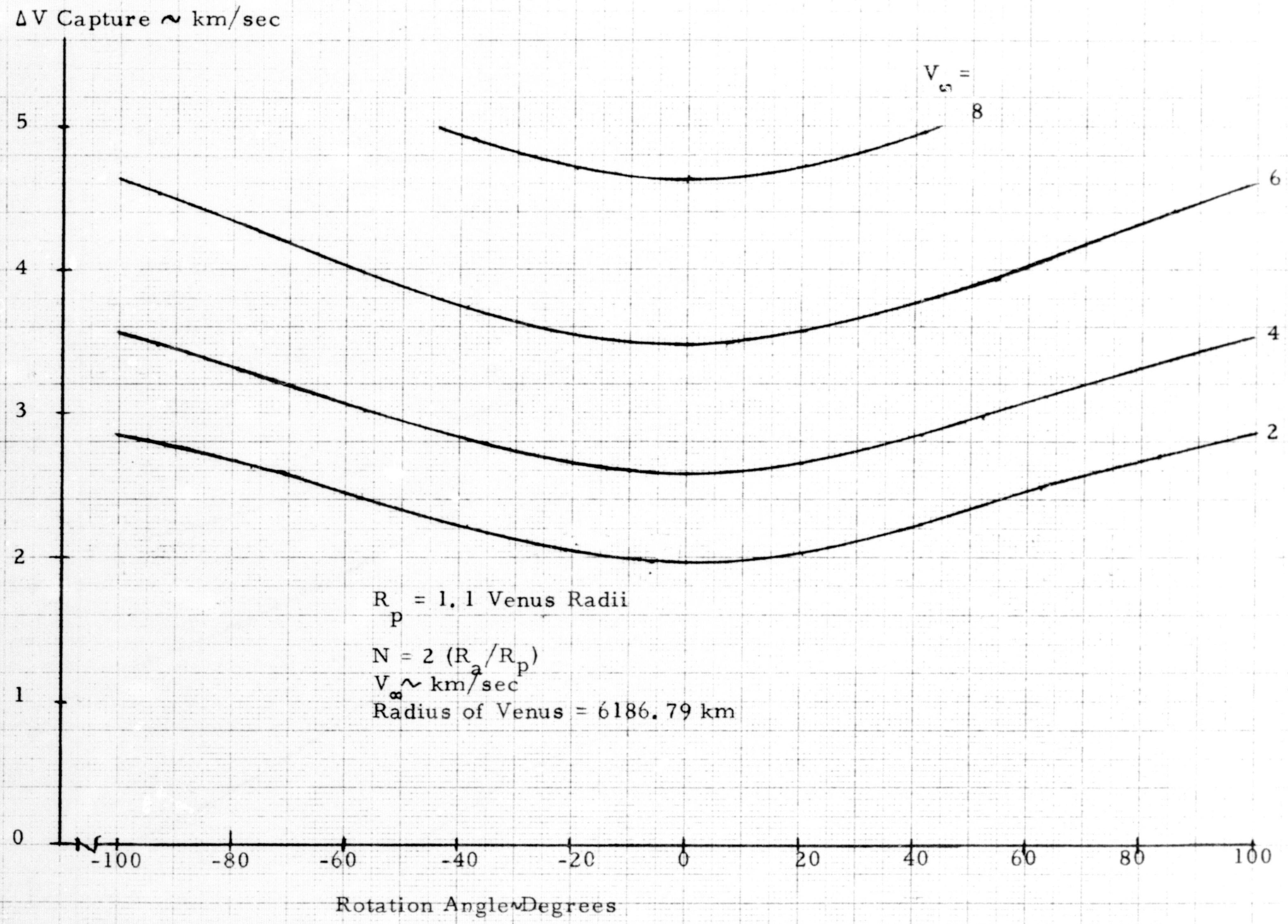
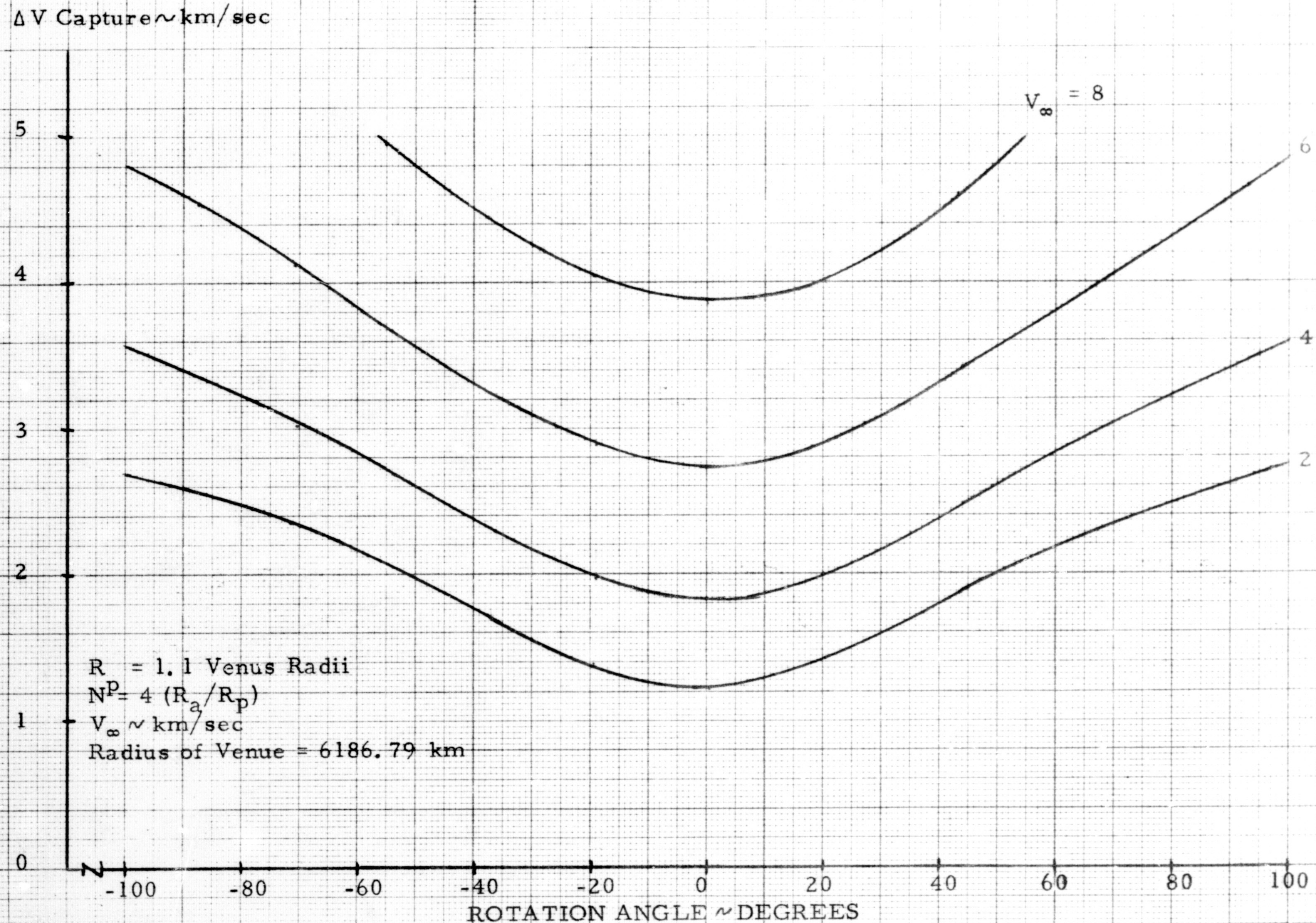


FIGURE 3. IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT VENUS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER



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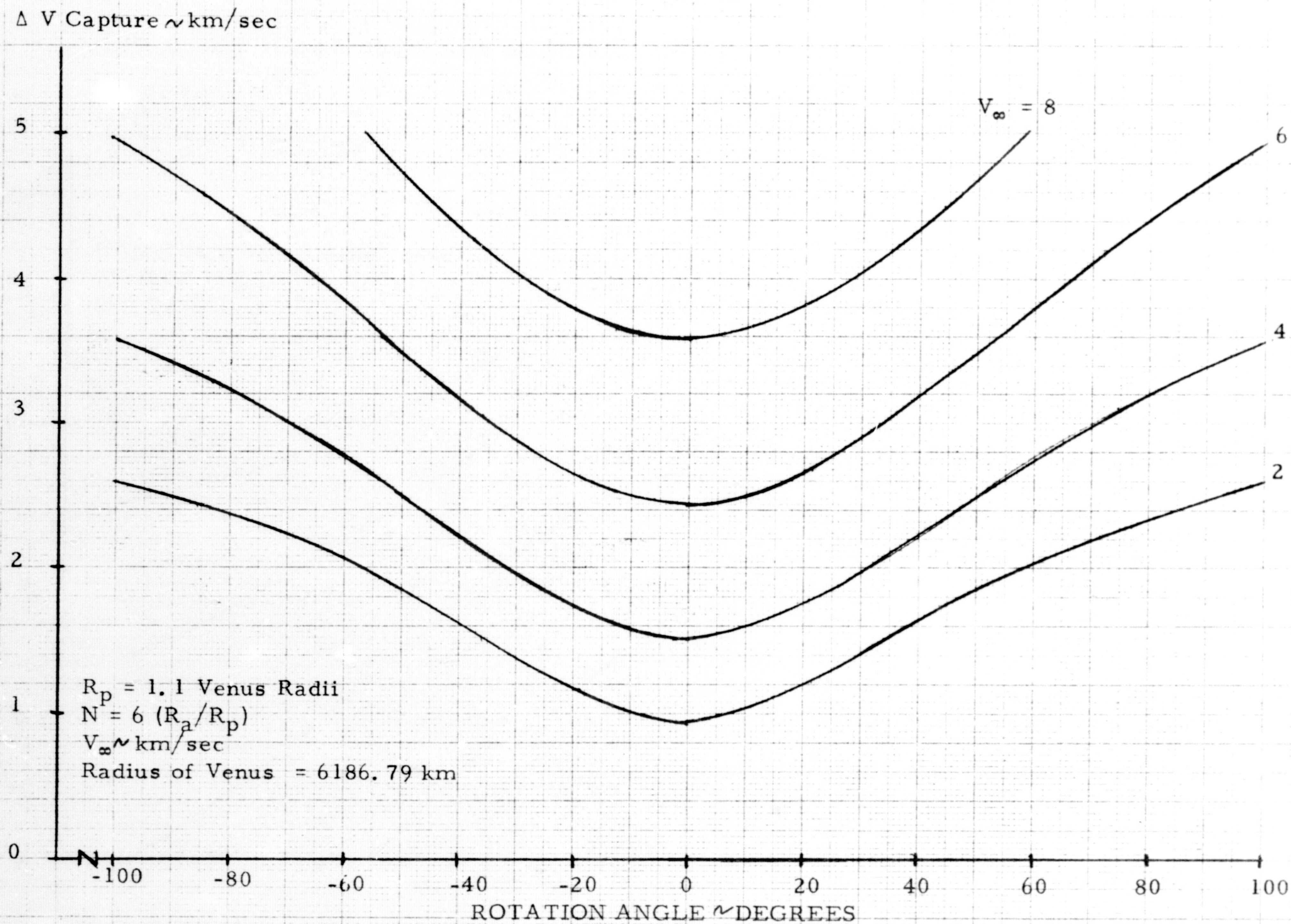

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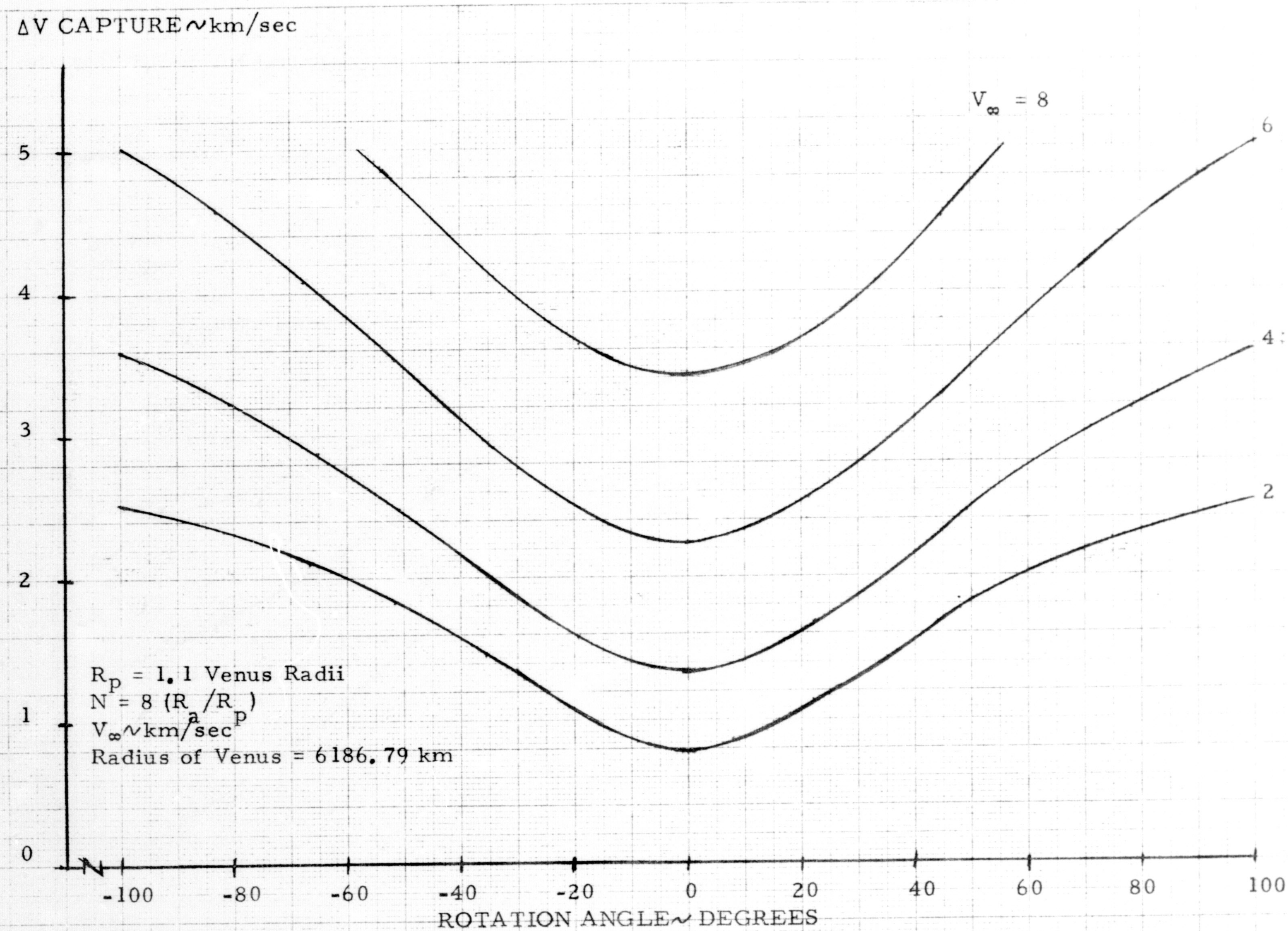

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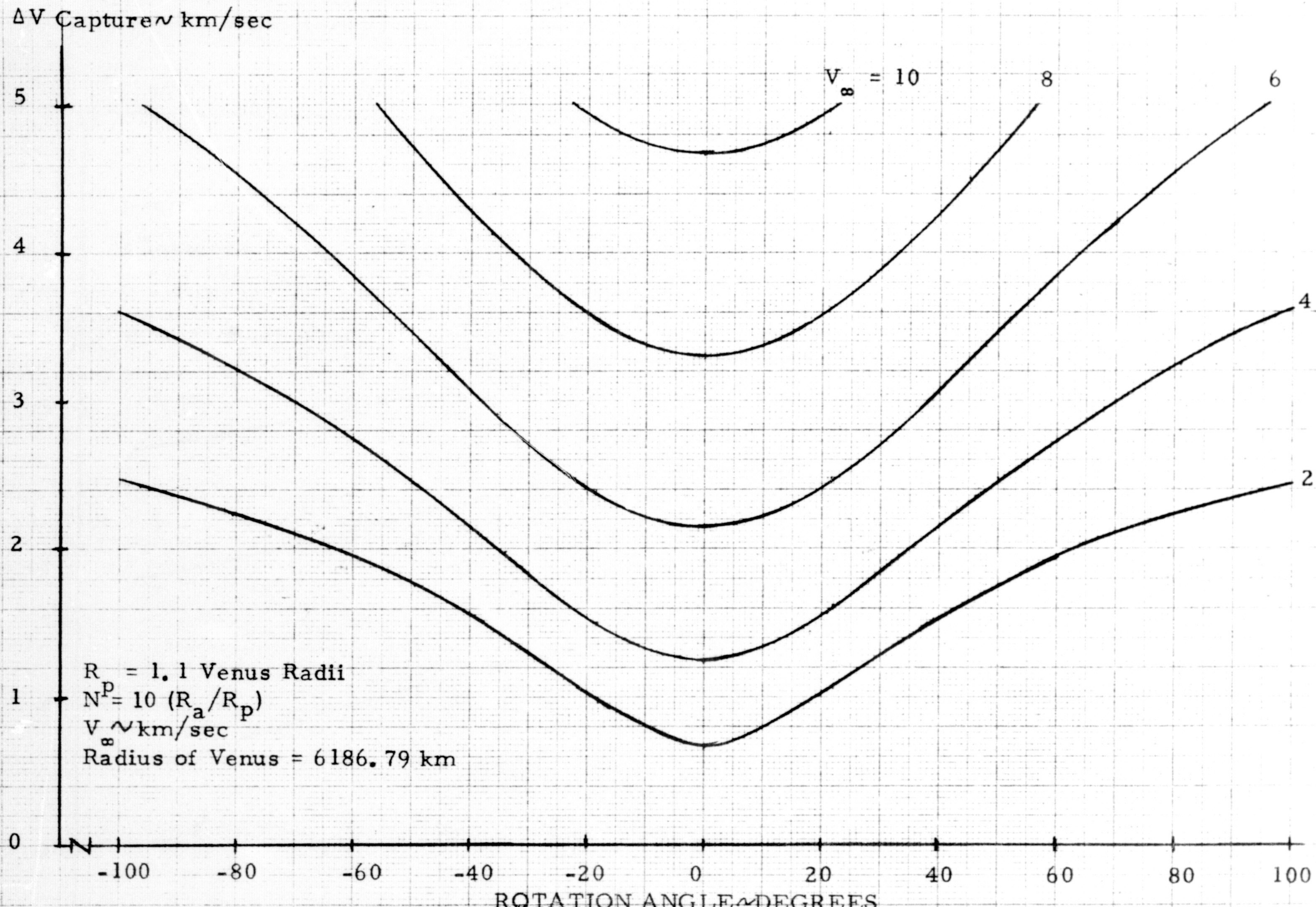

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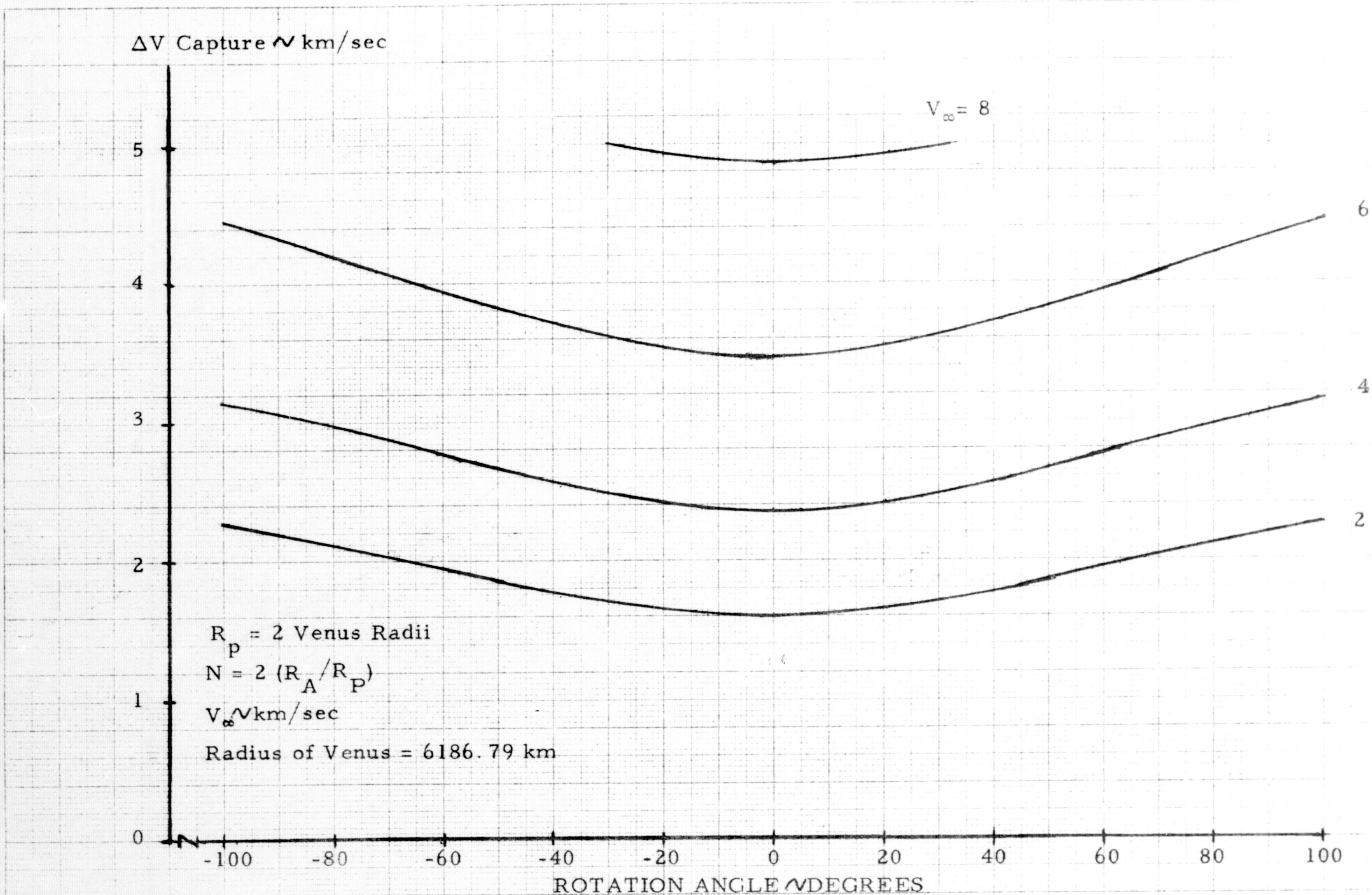

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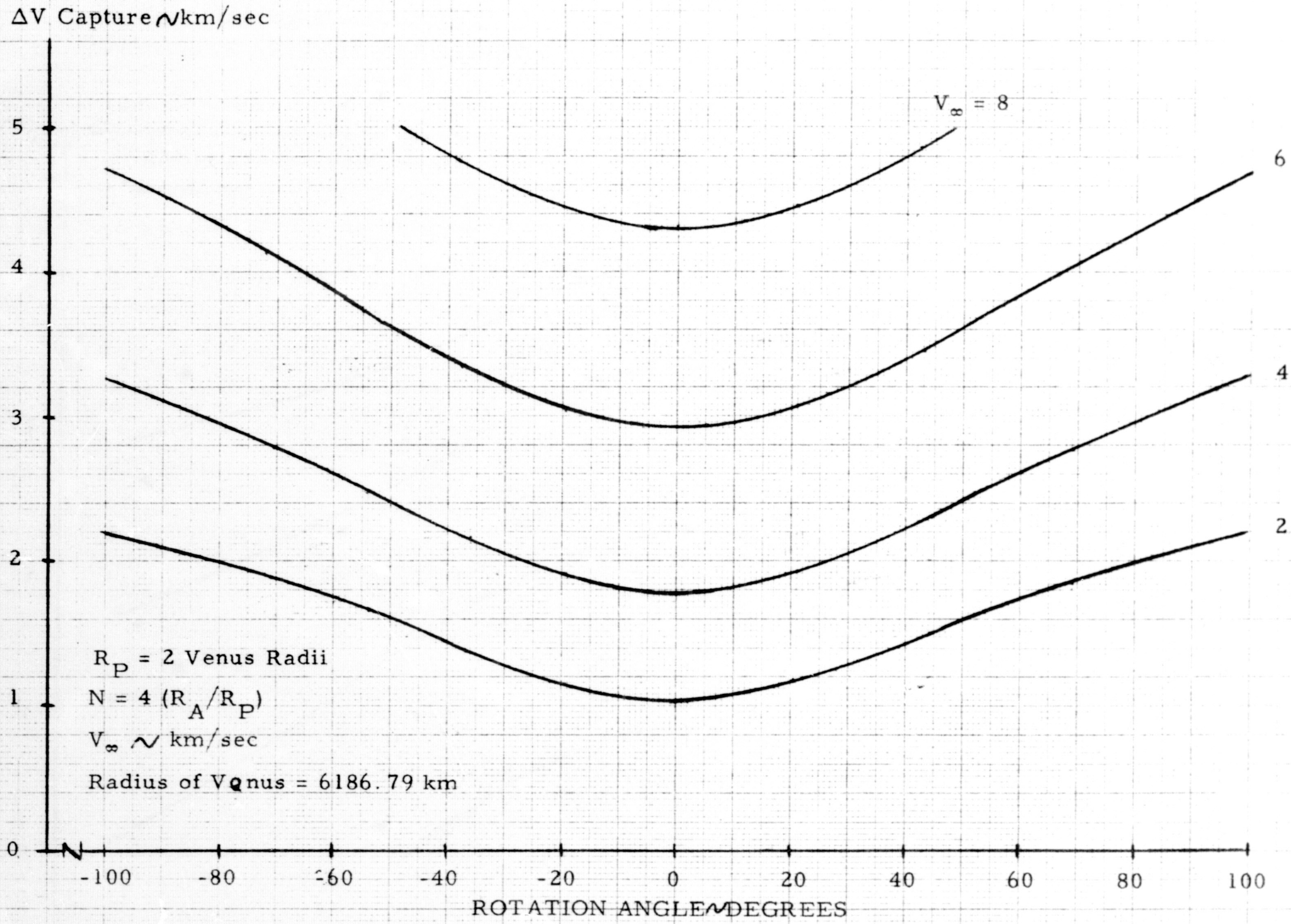

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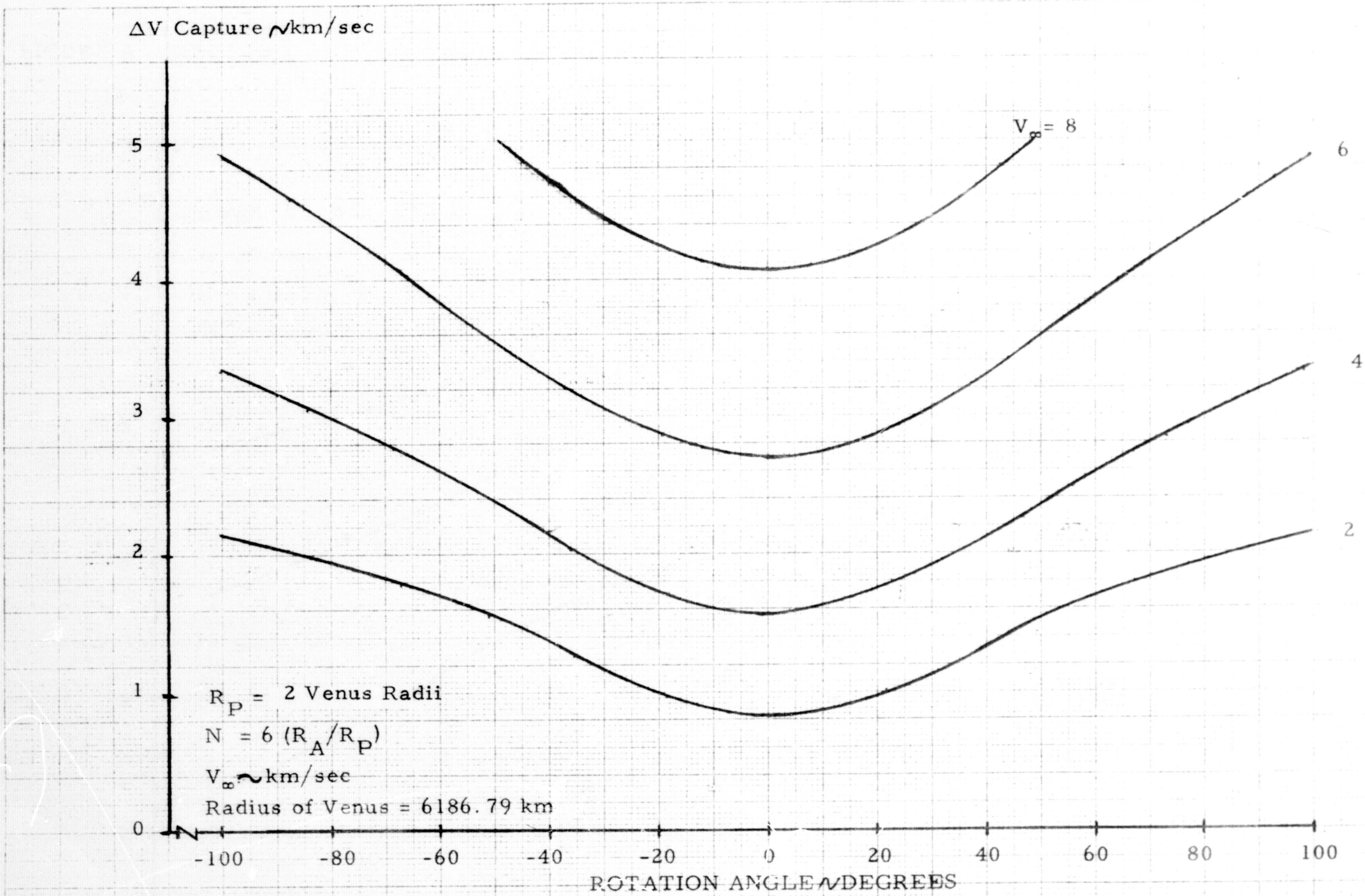

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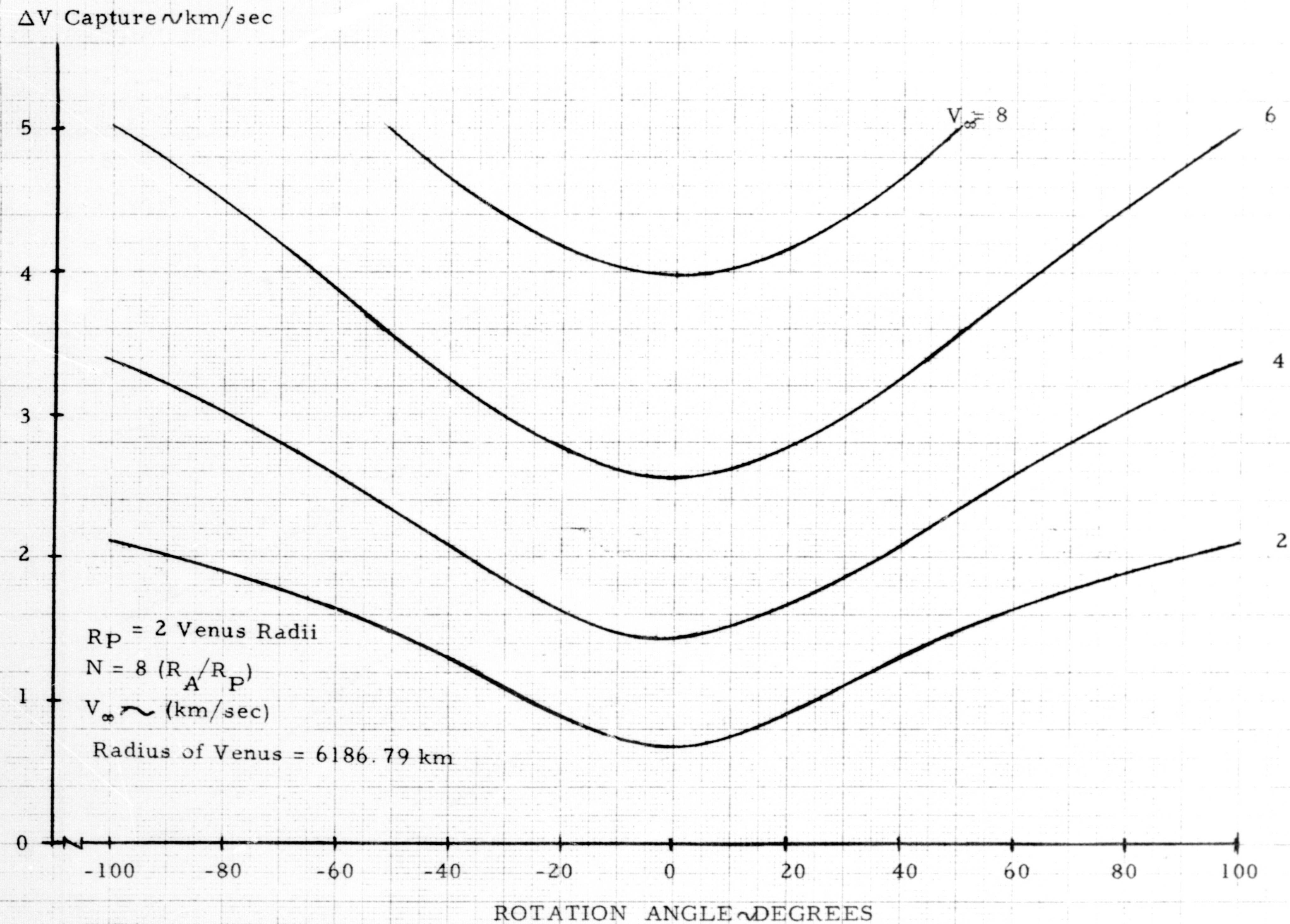

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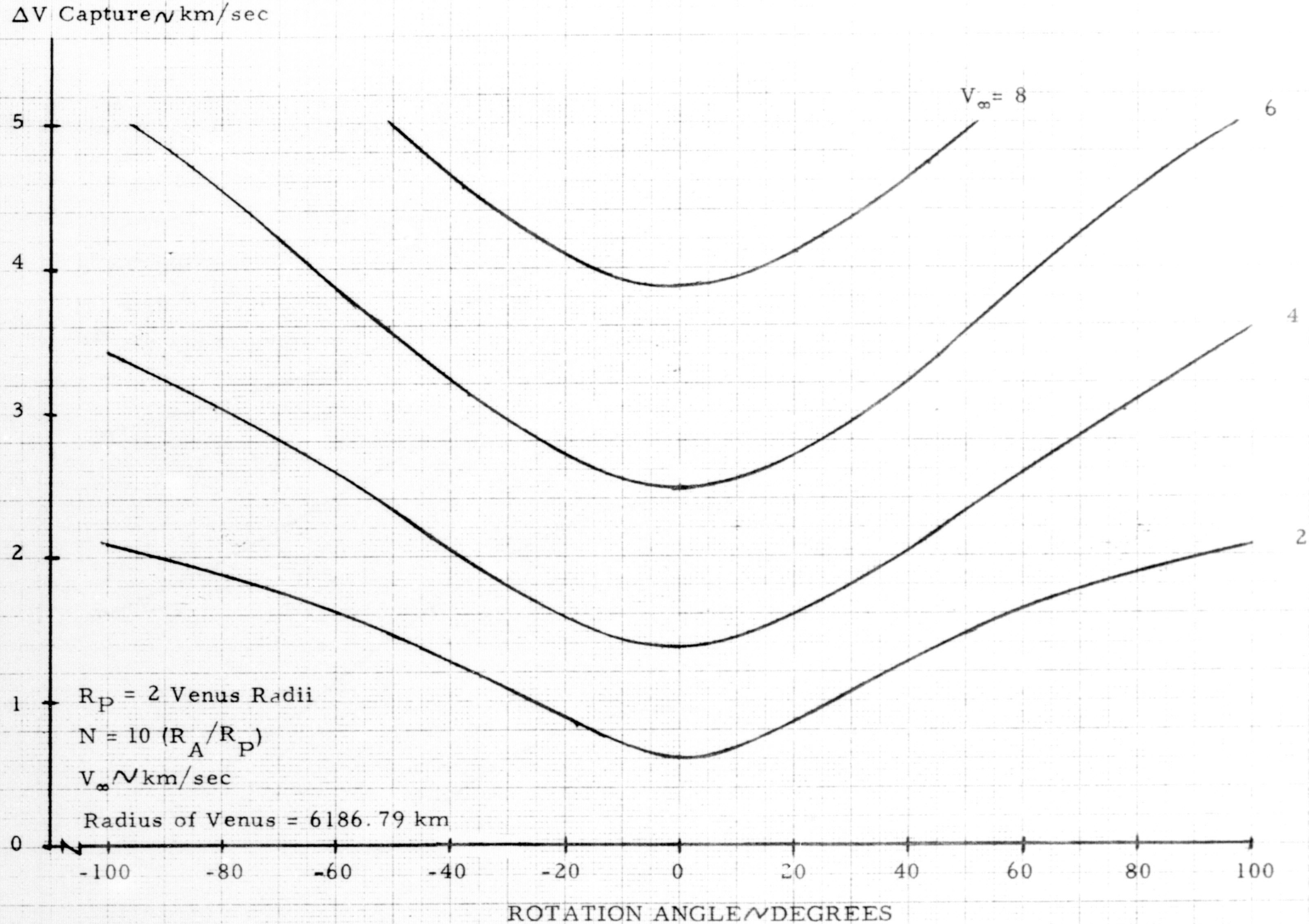

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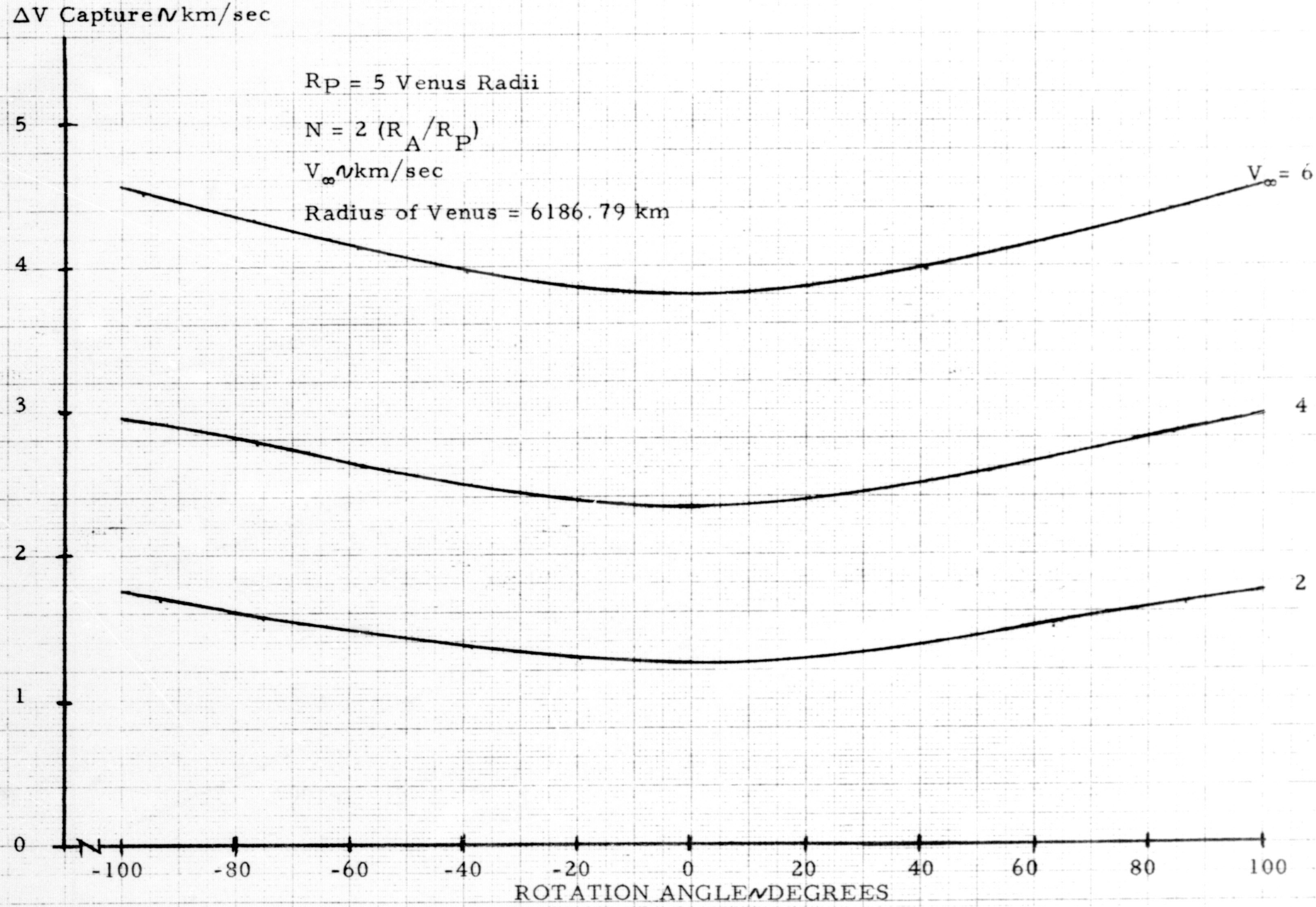

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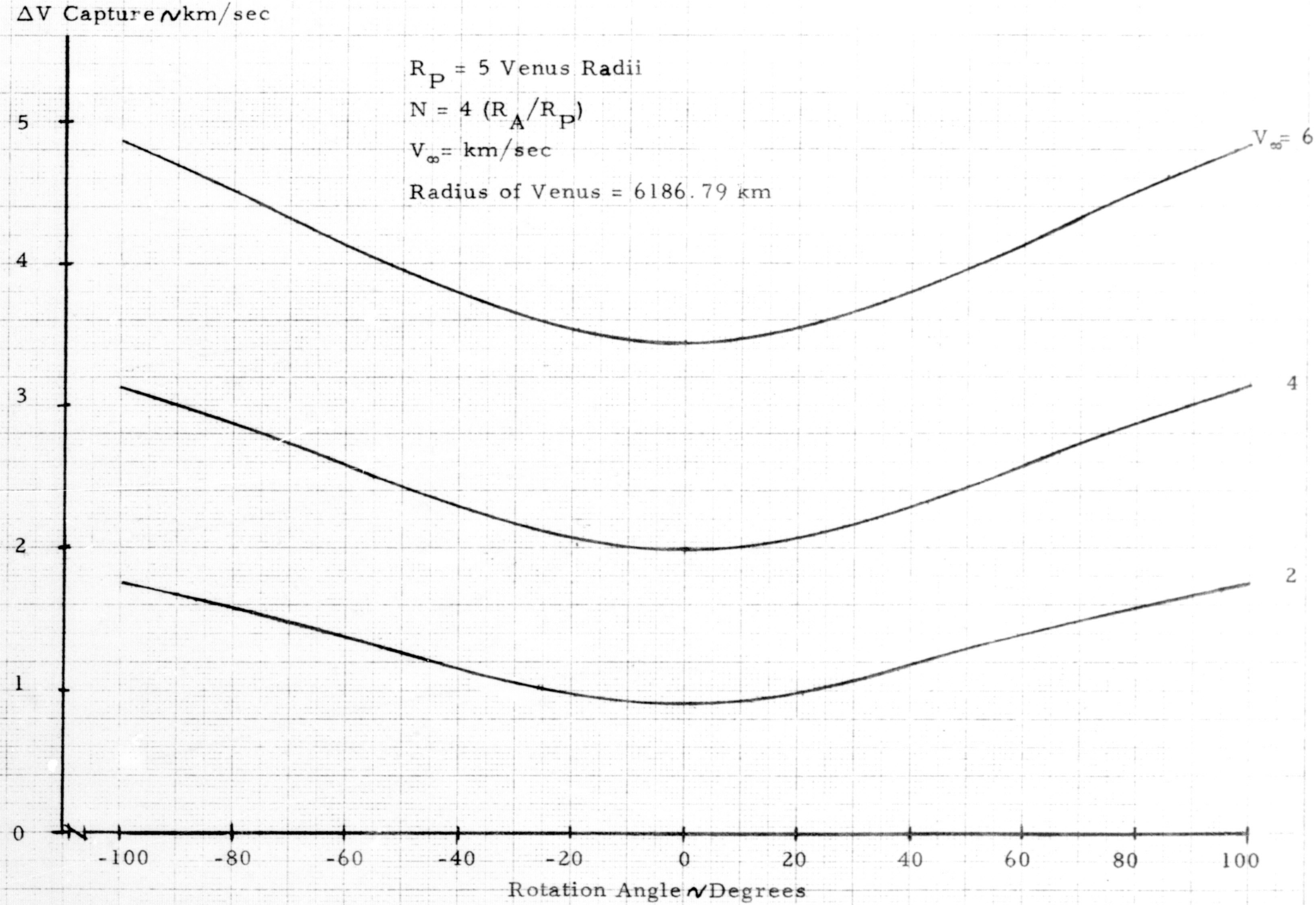

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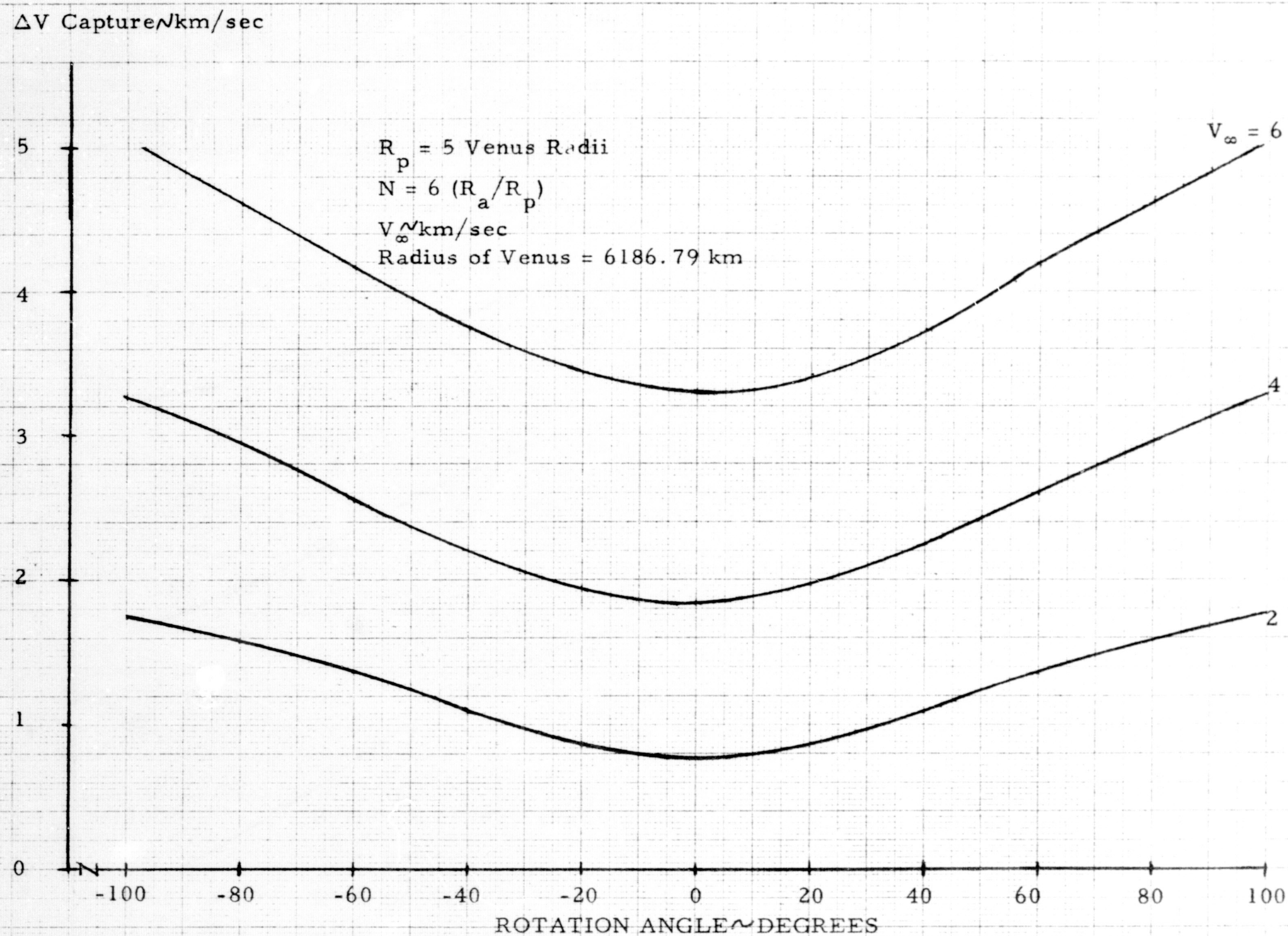

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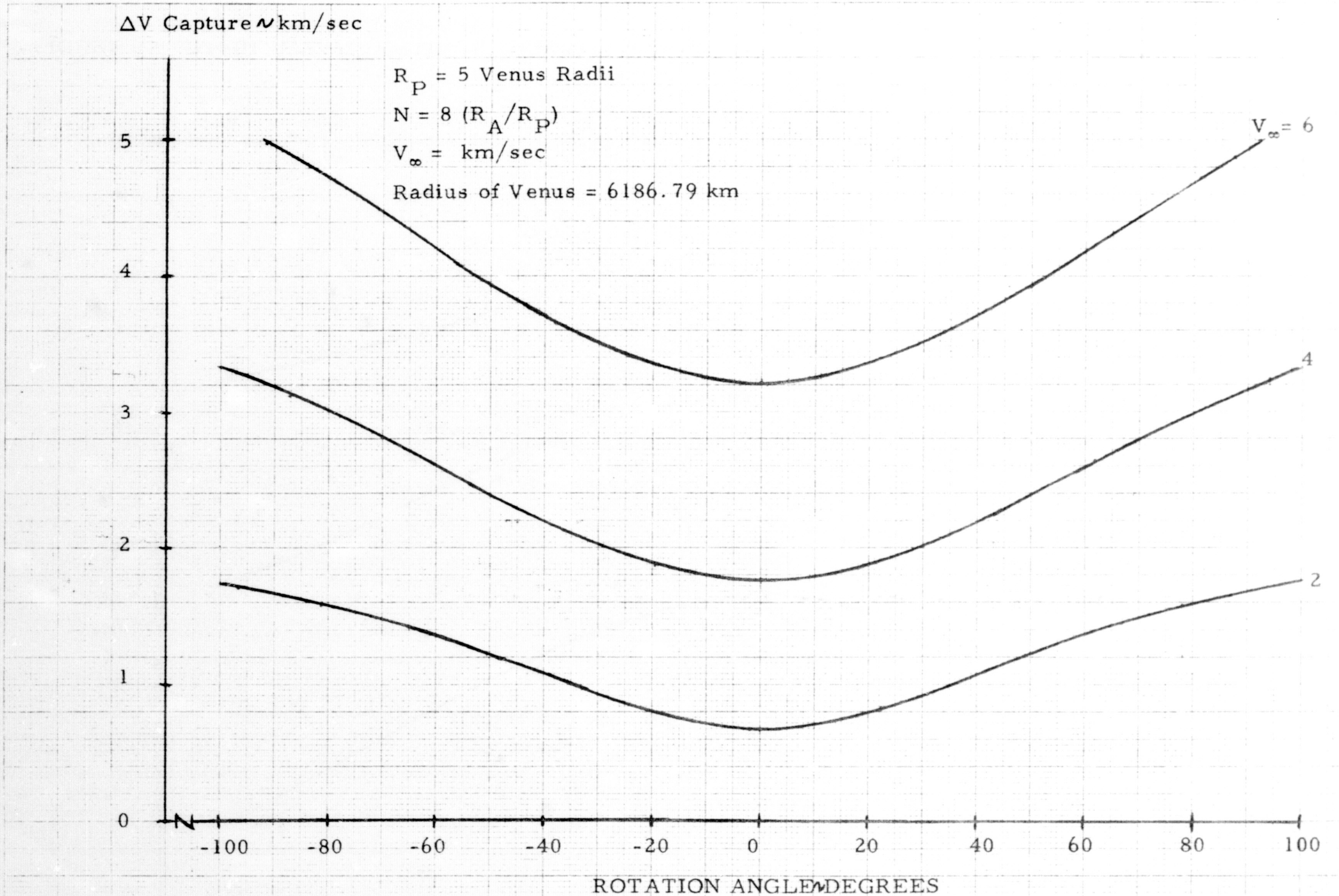

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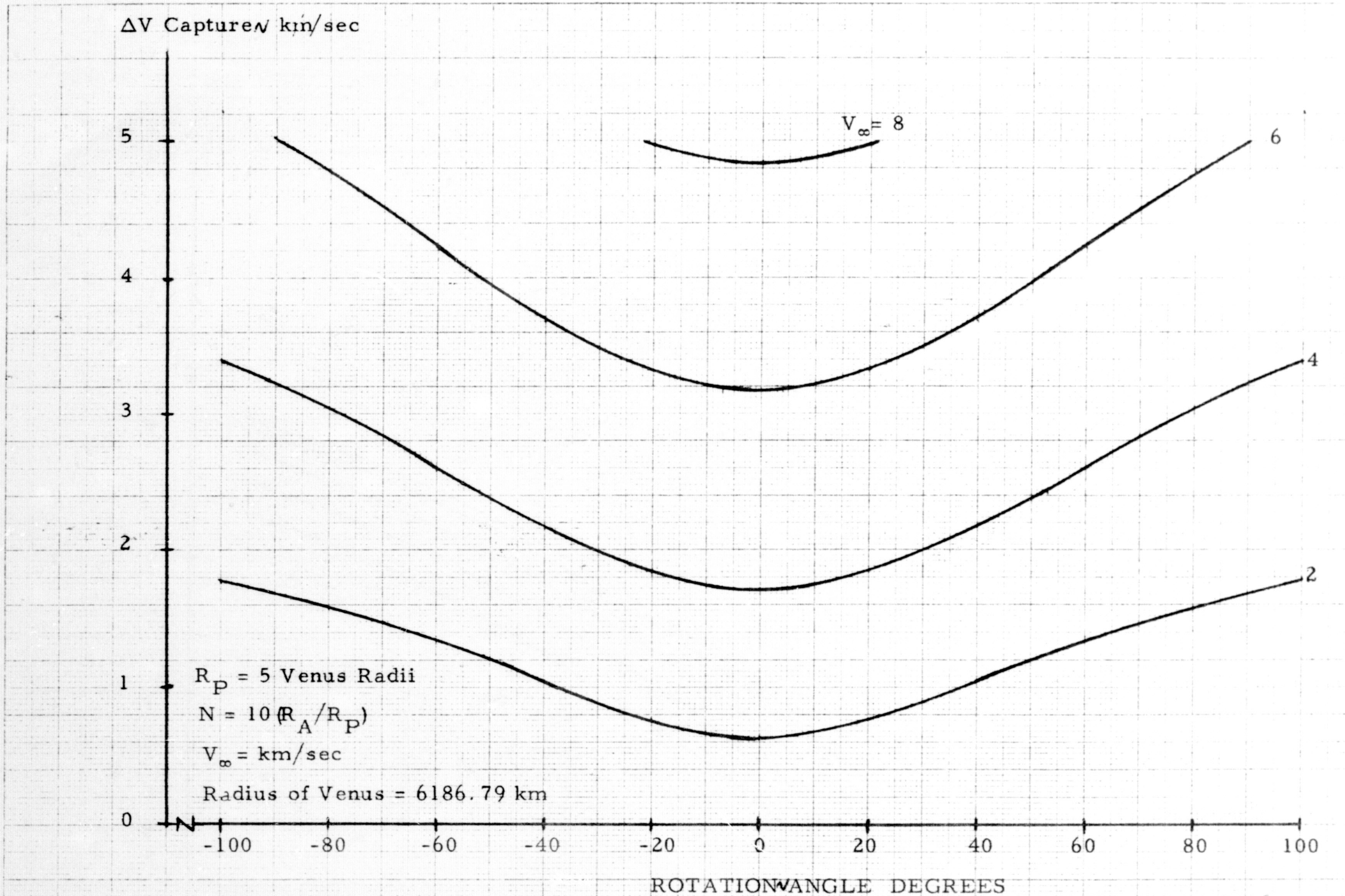

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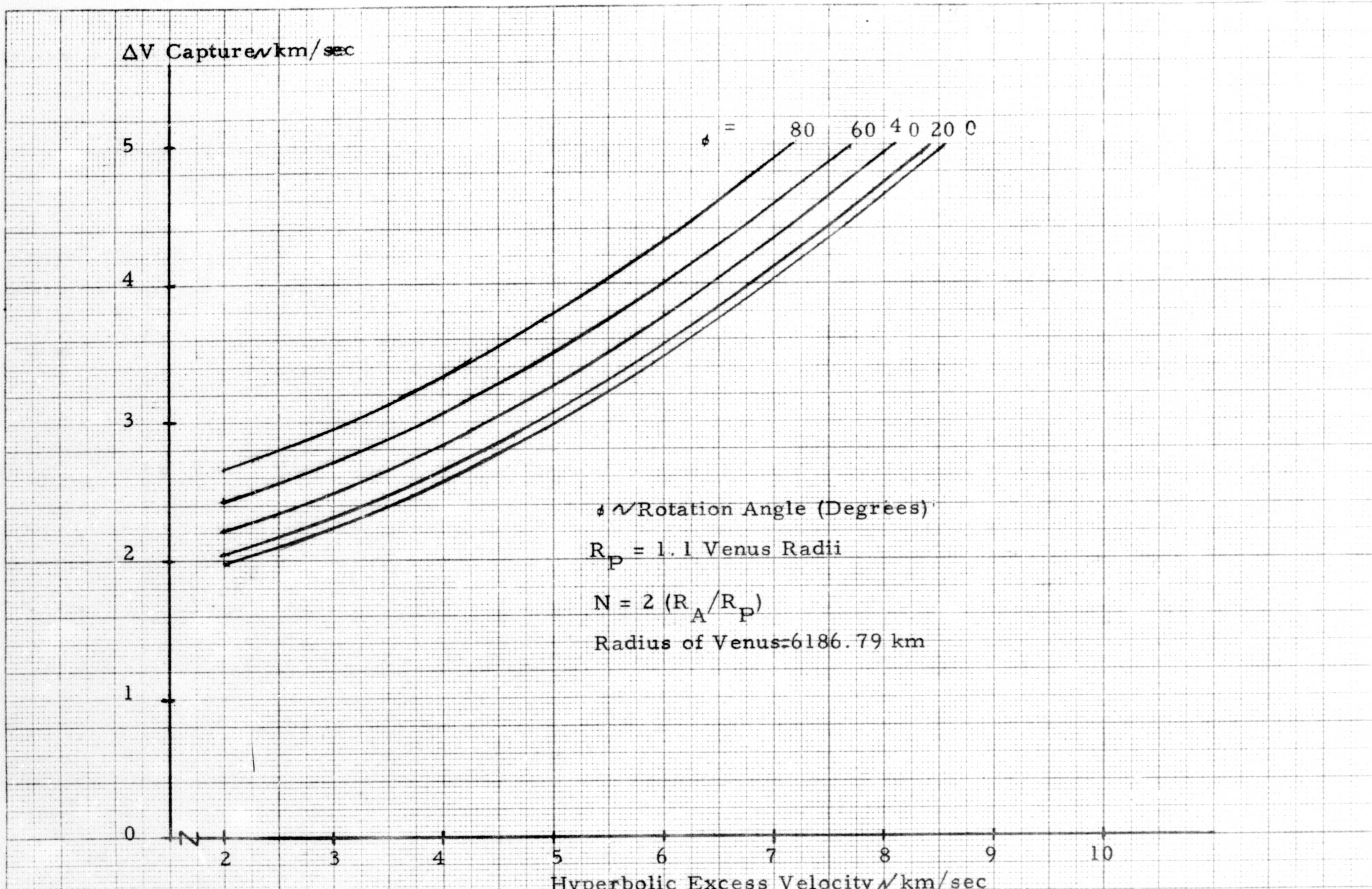
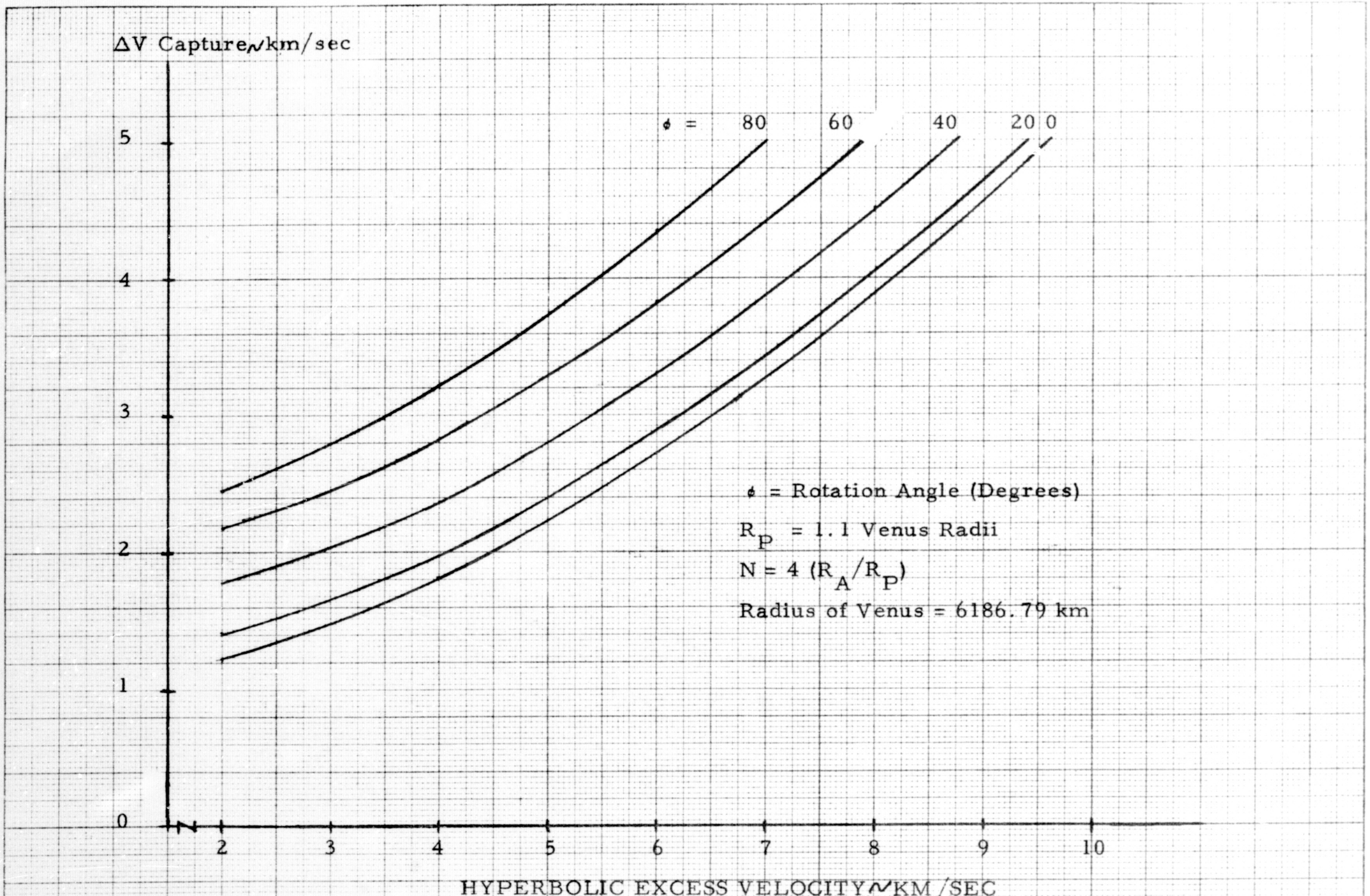


Figure 18 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT VENUS AS FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) for TANGENTIAL TRANSFER



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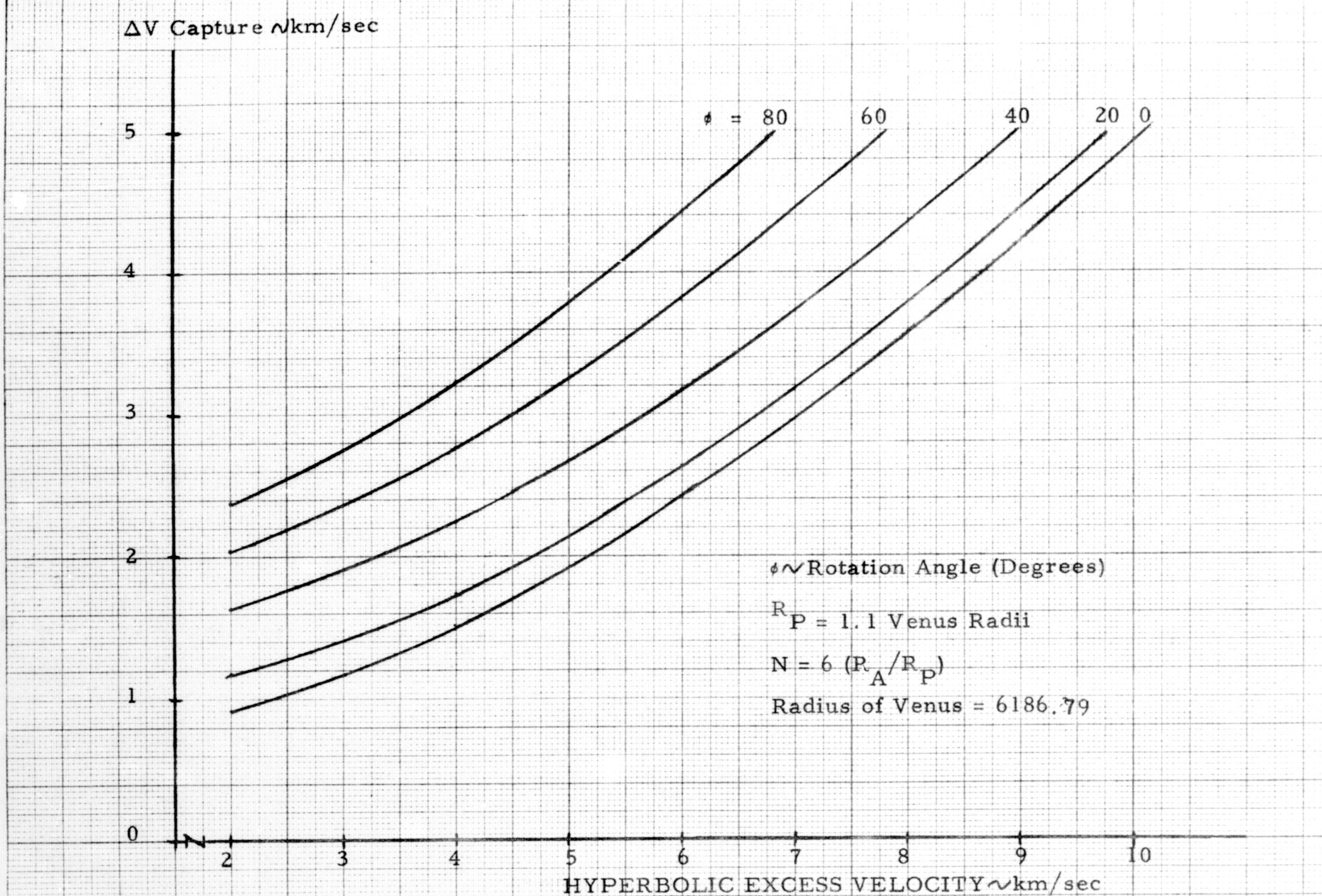

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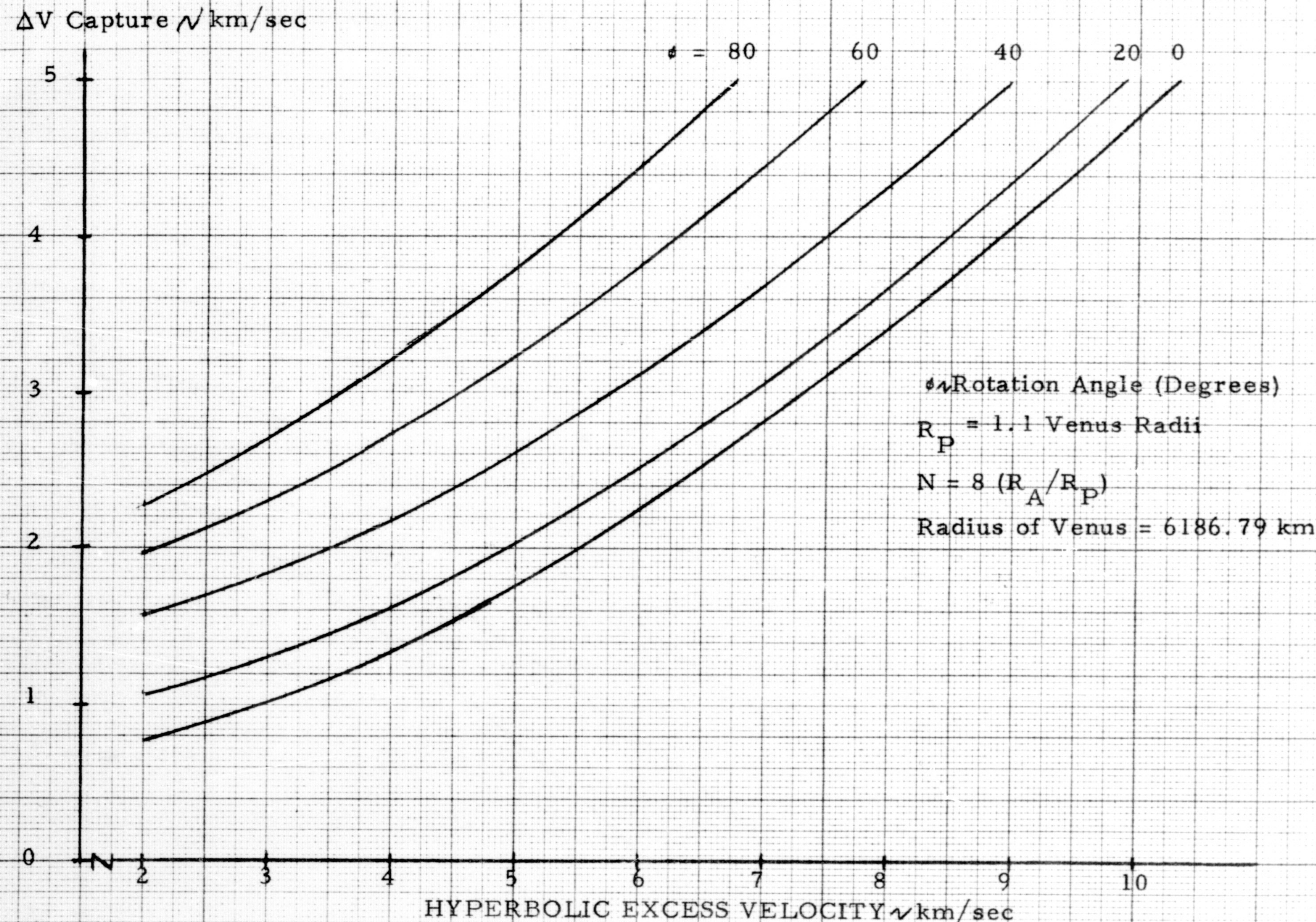

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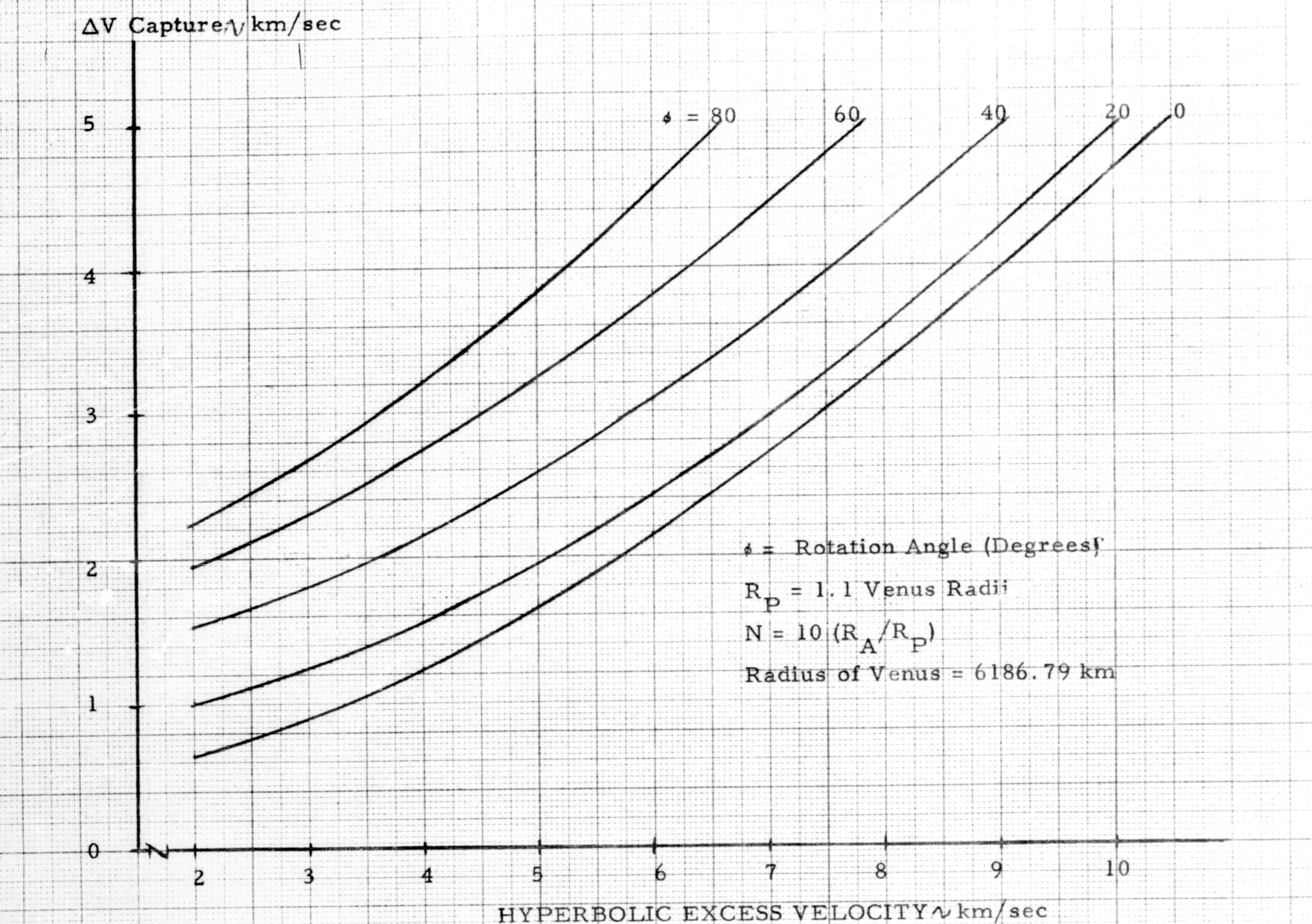

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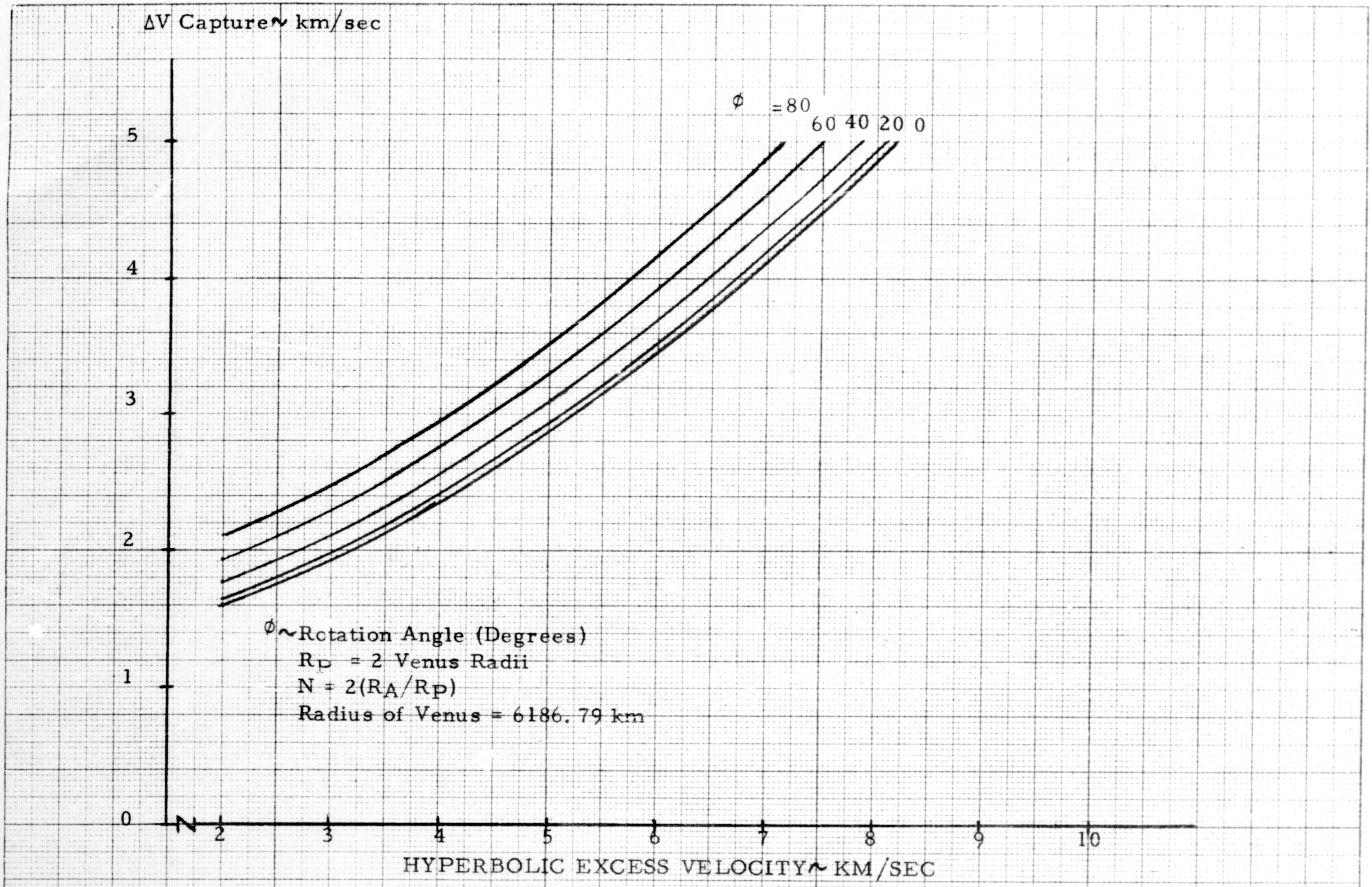

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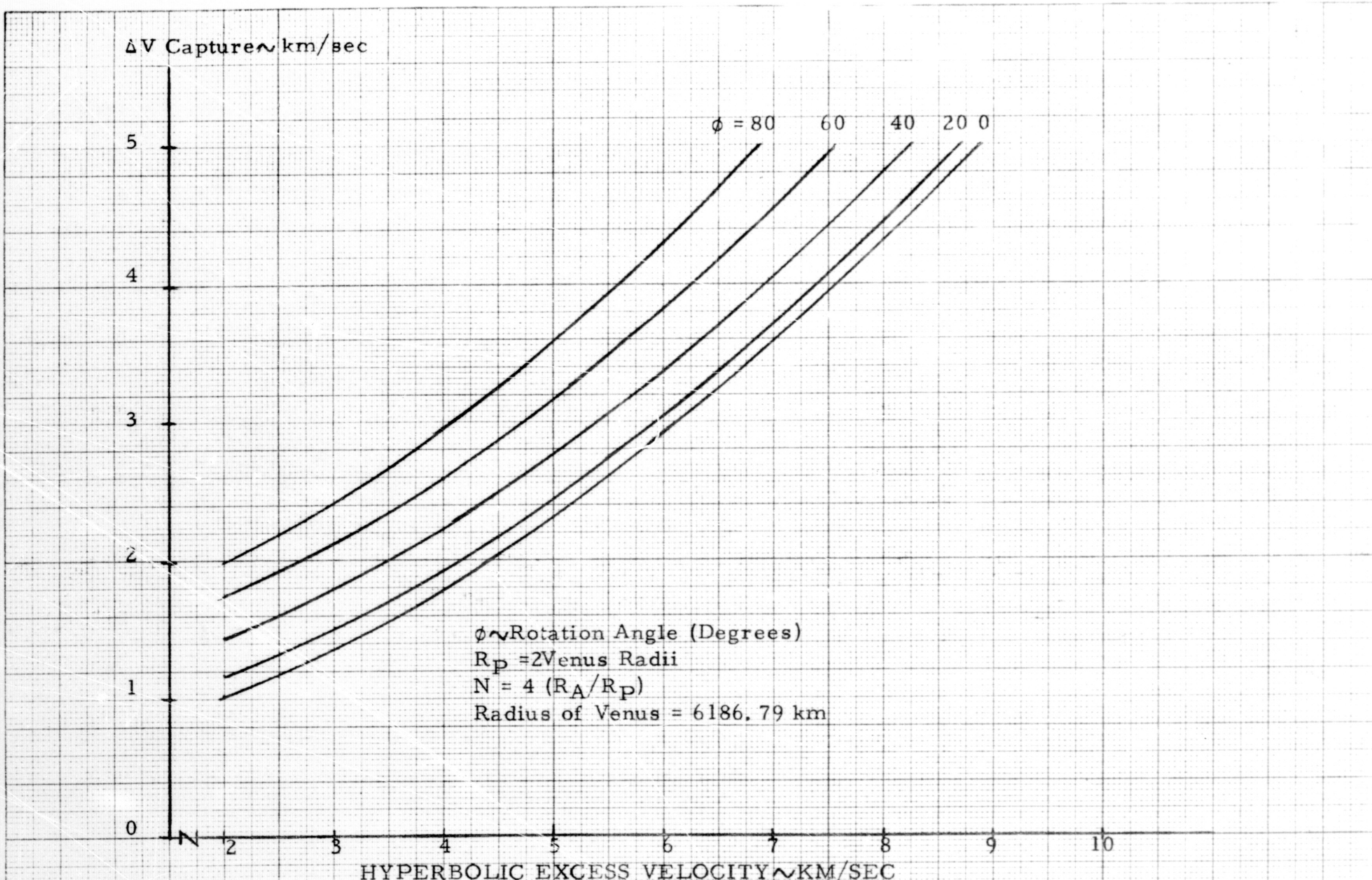

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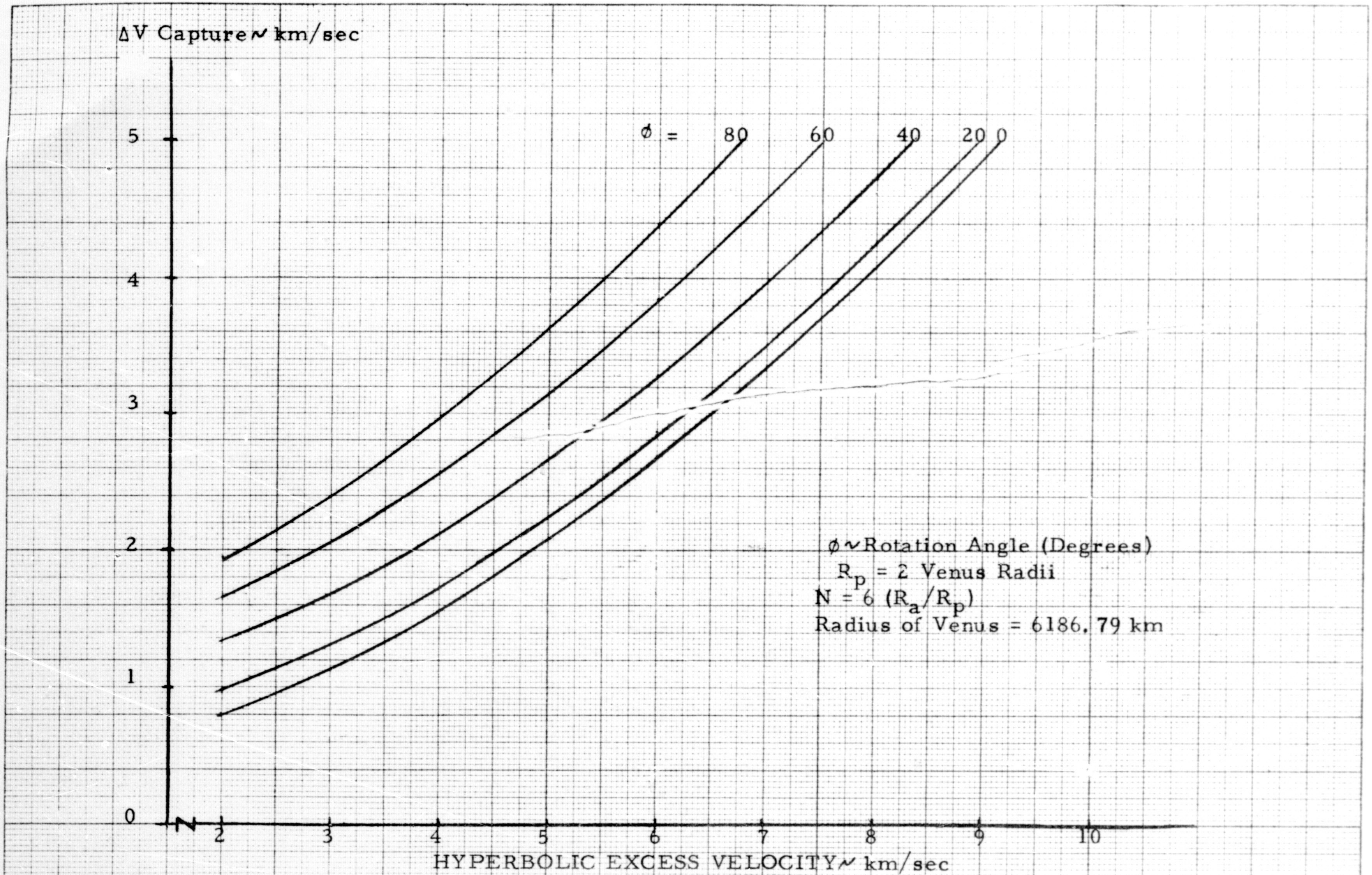

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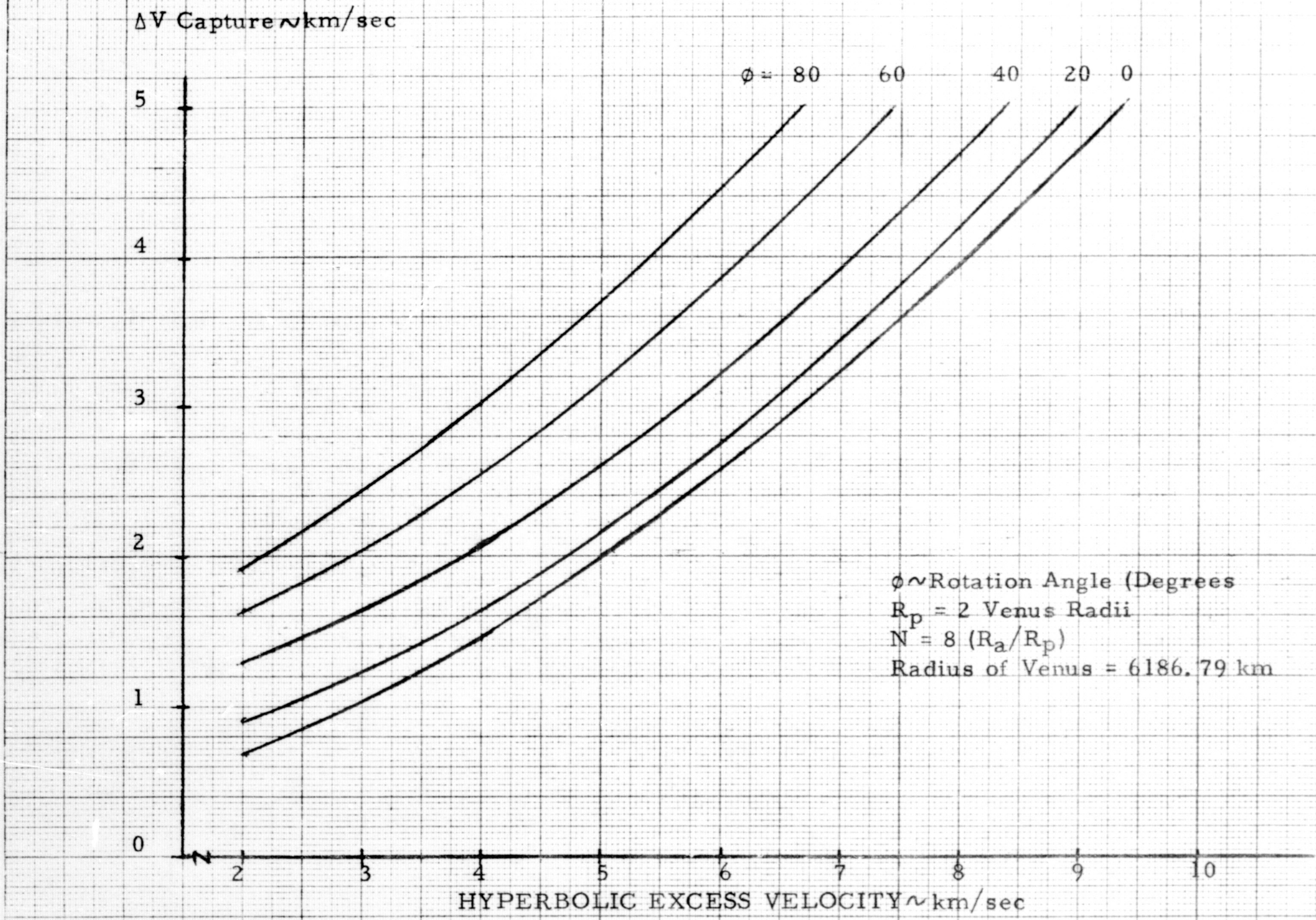

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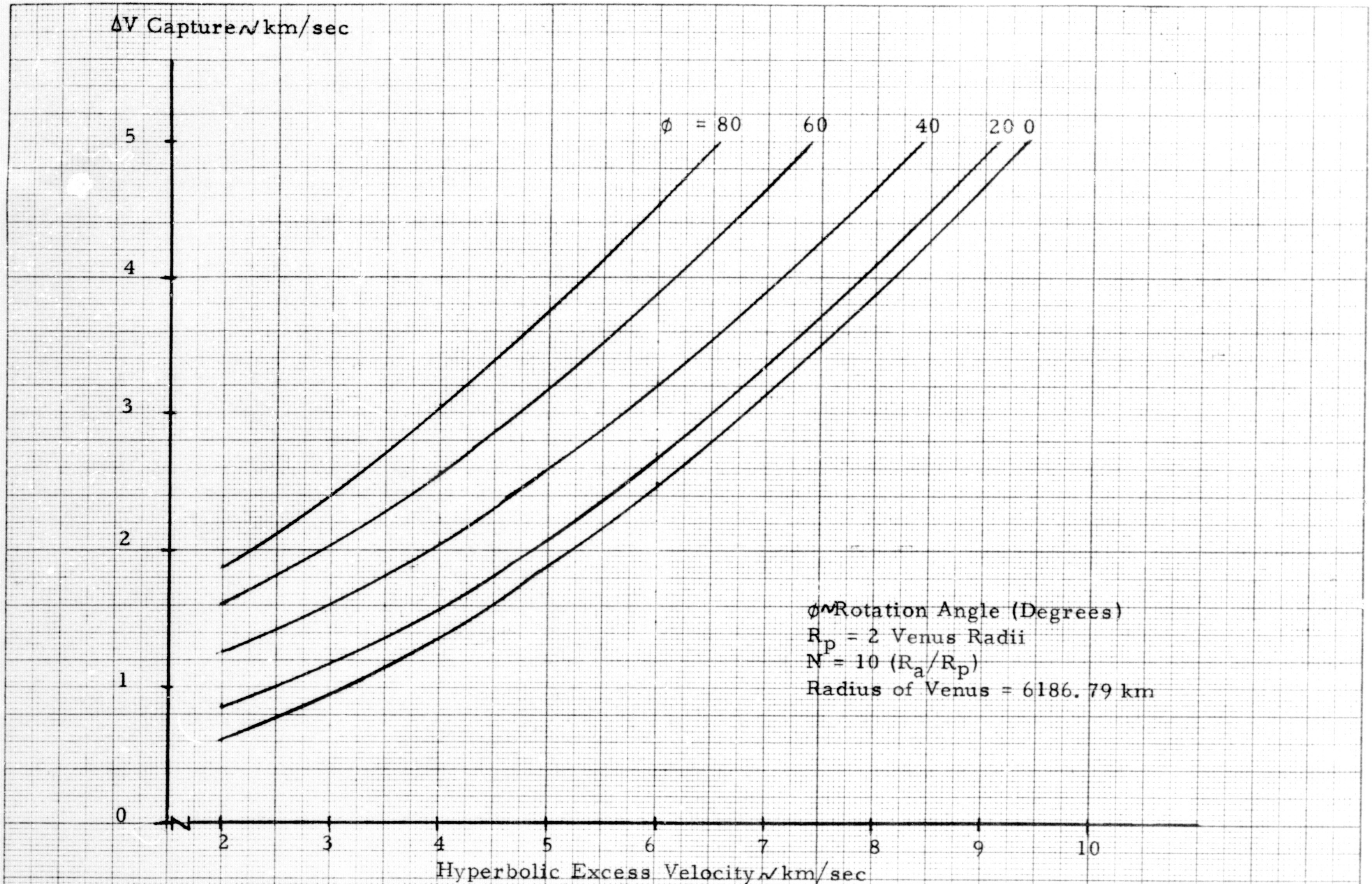

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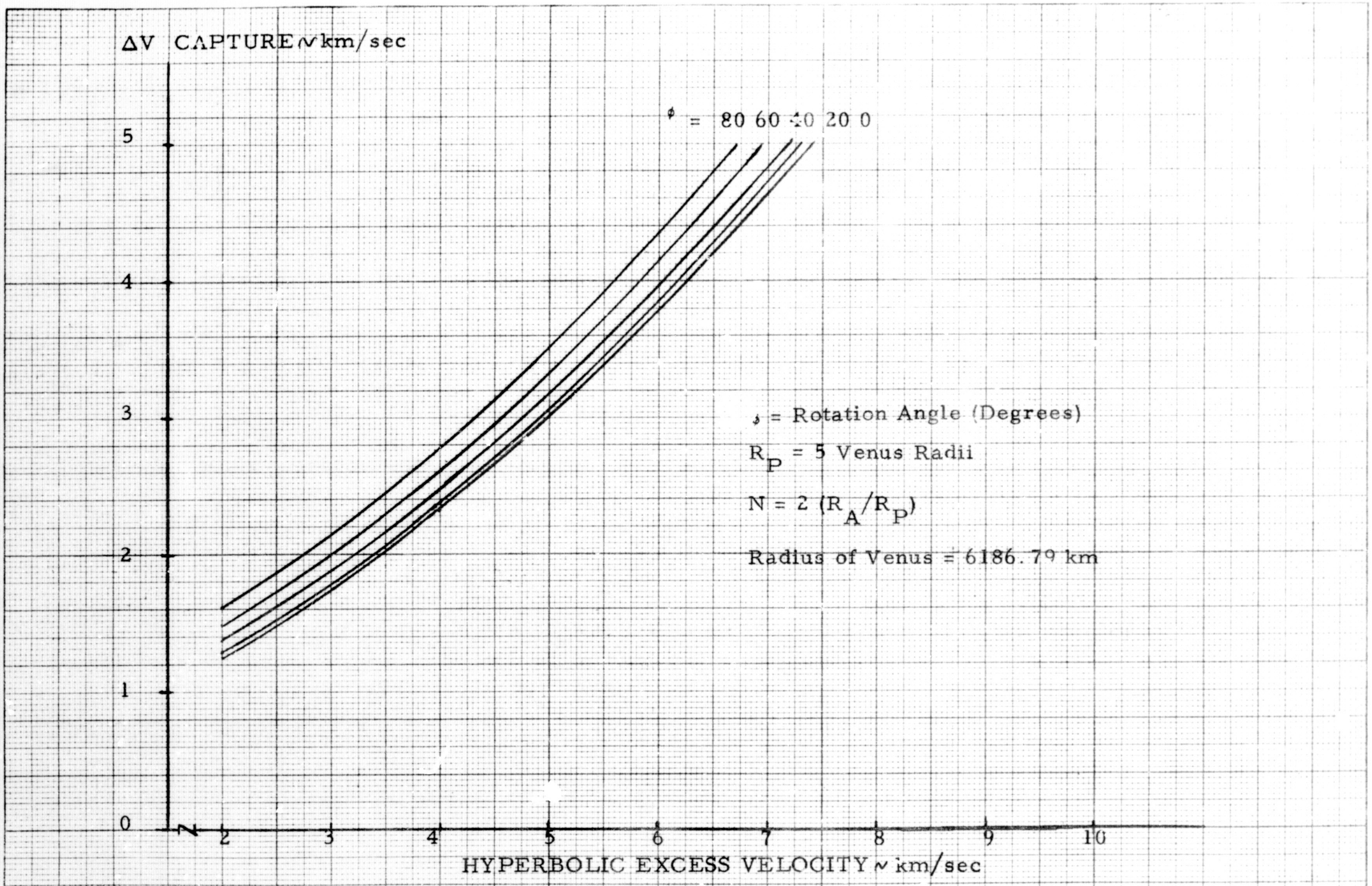

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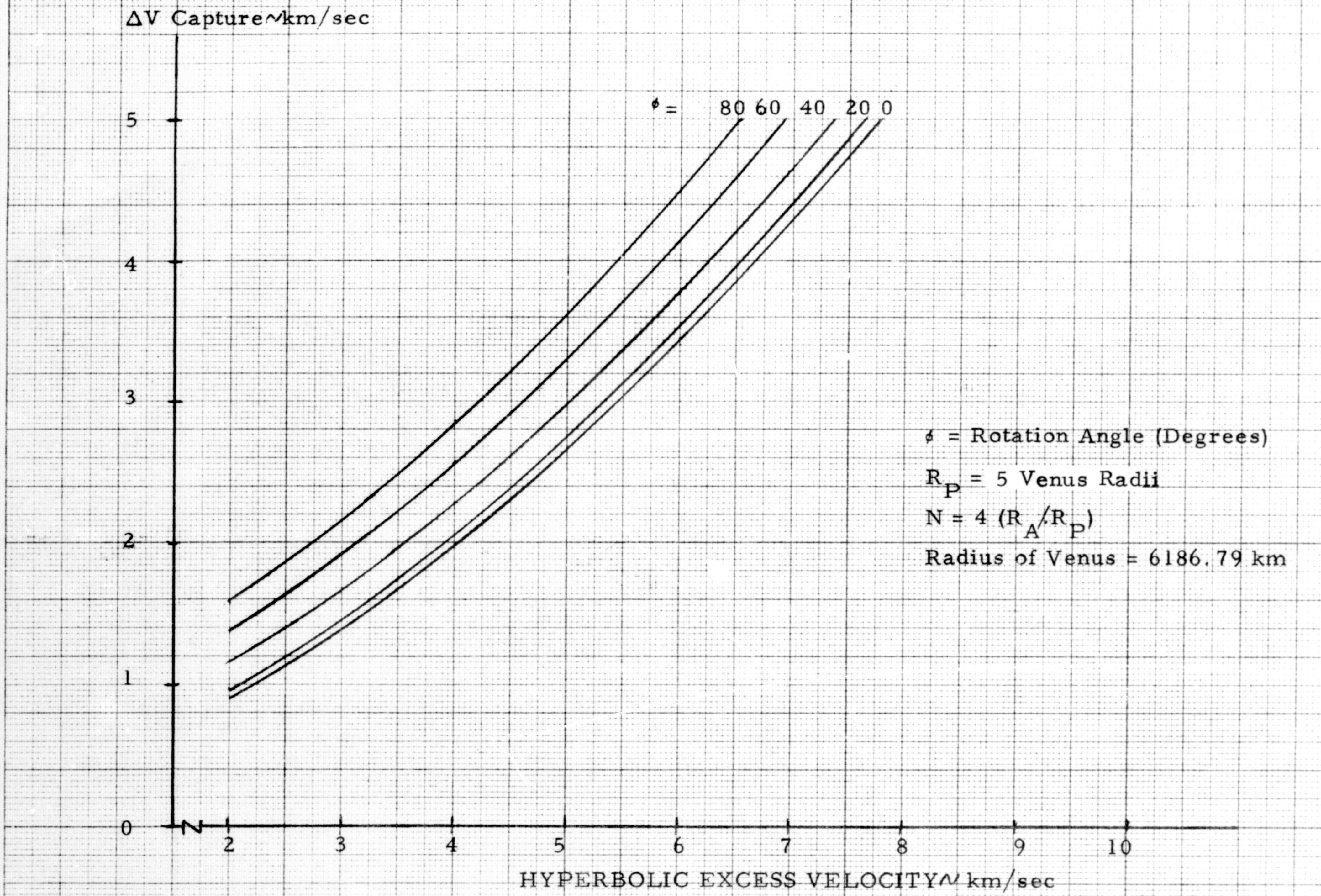

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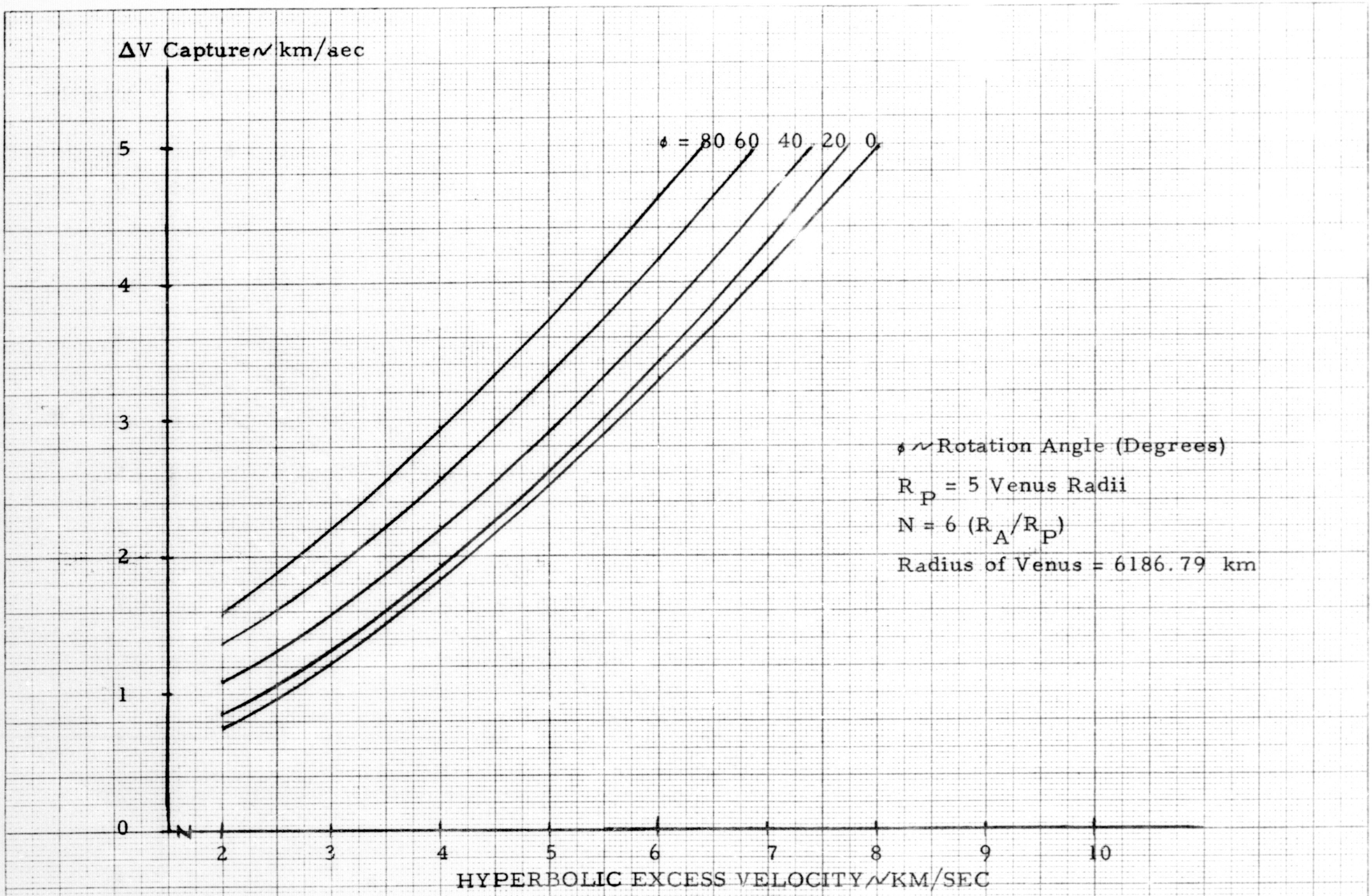

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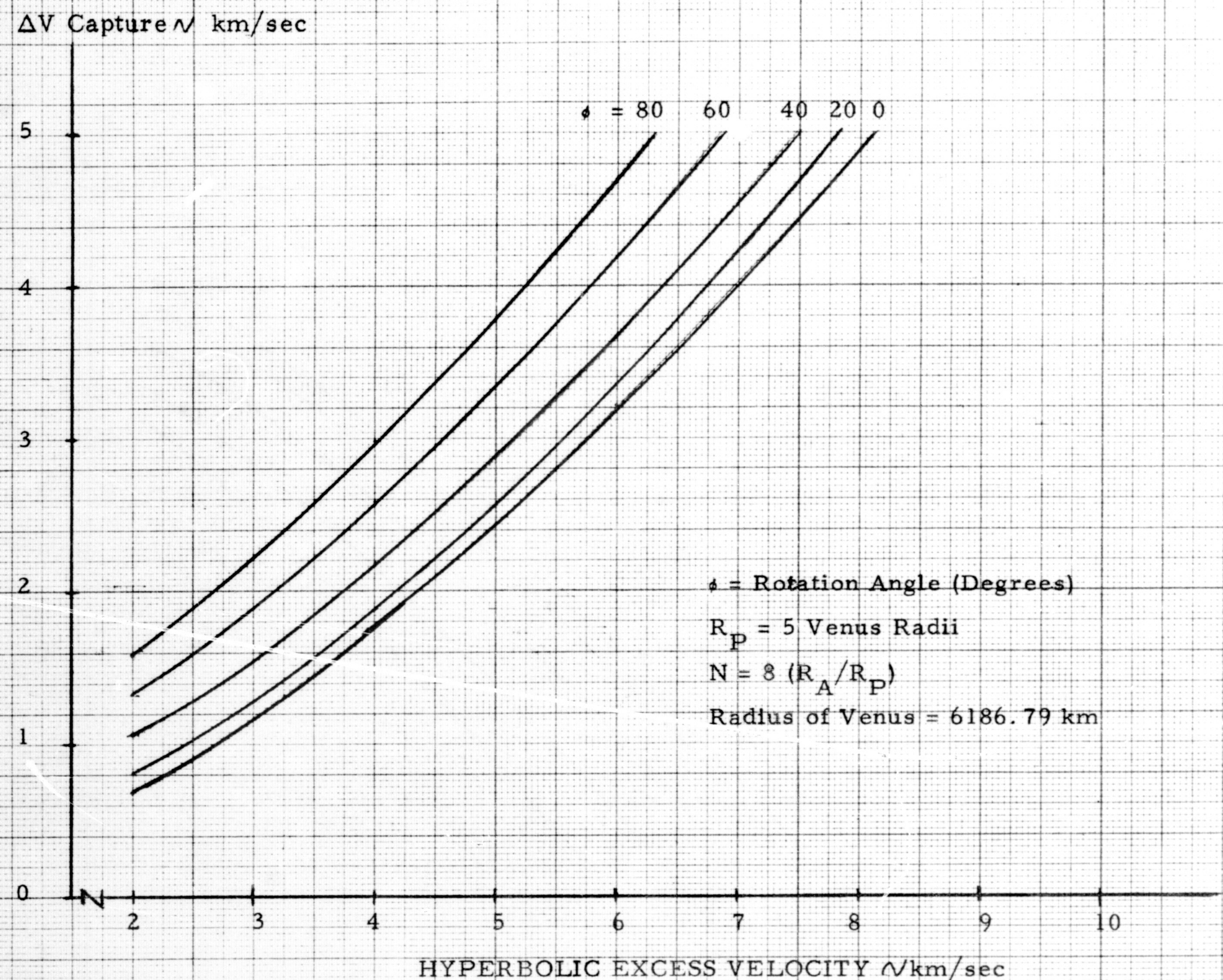

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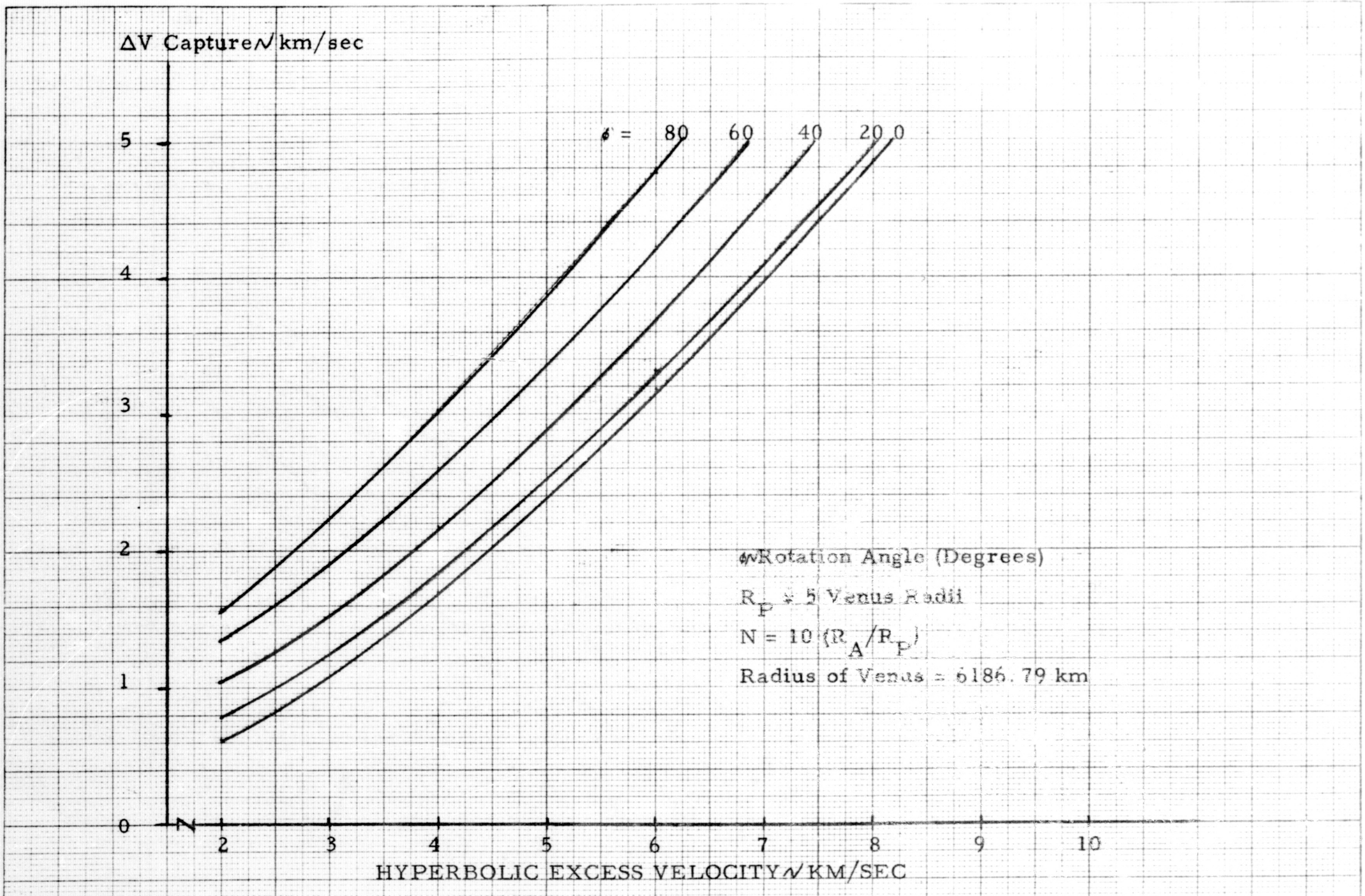

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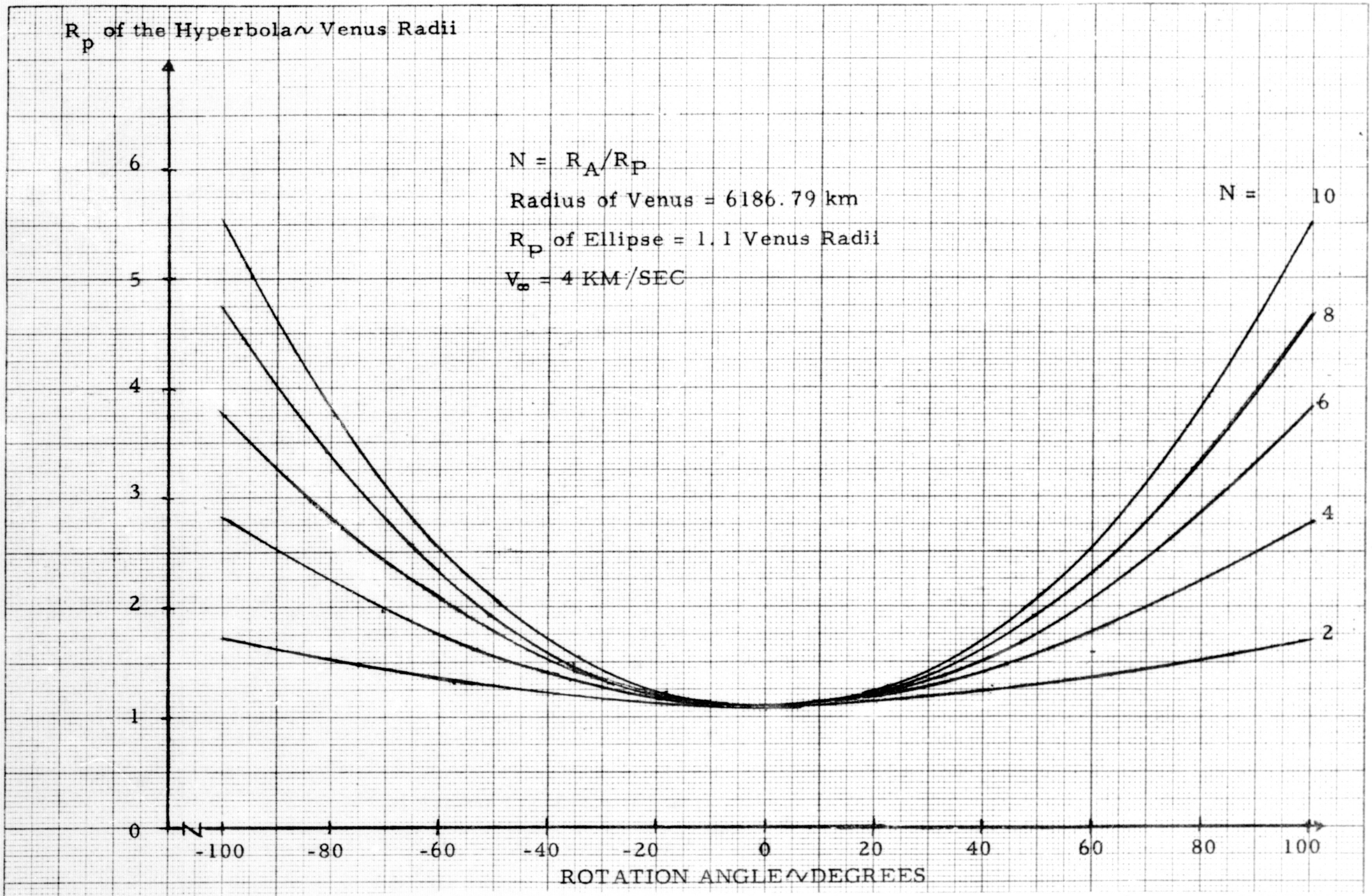


FIGURE 33 : PERIAPSIS OF THE INCOMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO VENUS ORBIT

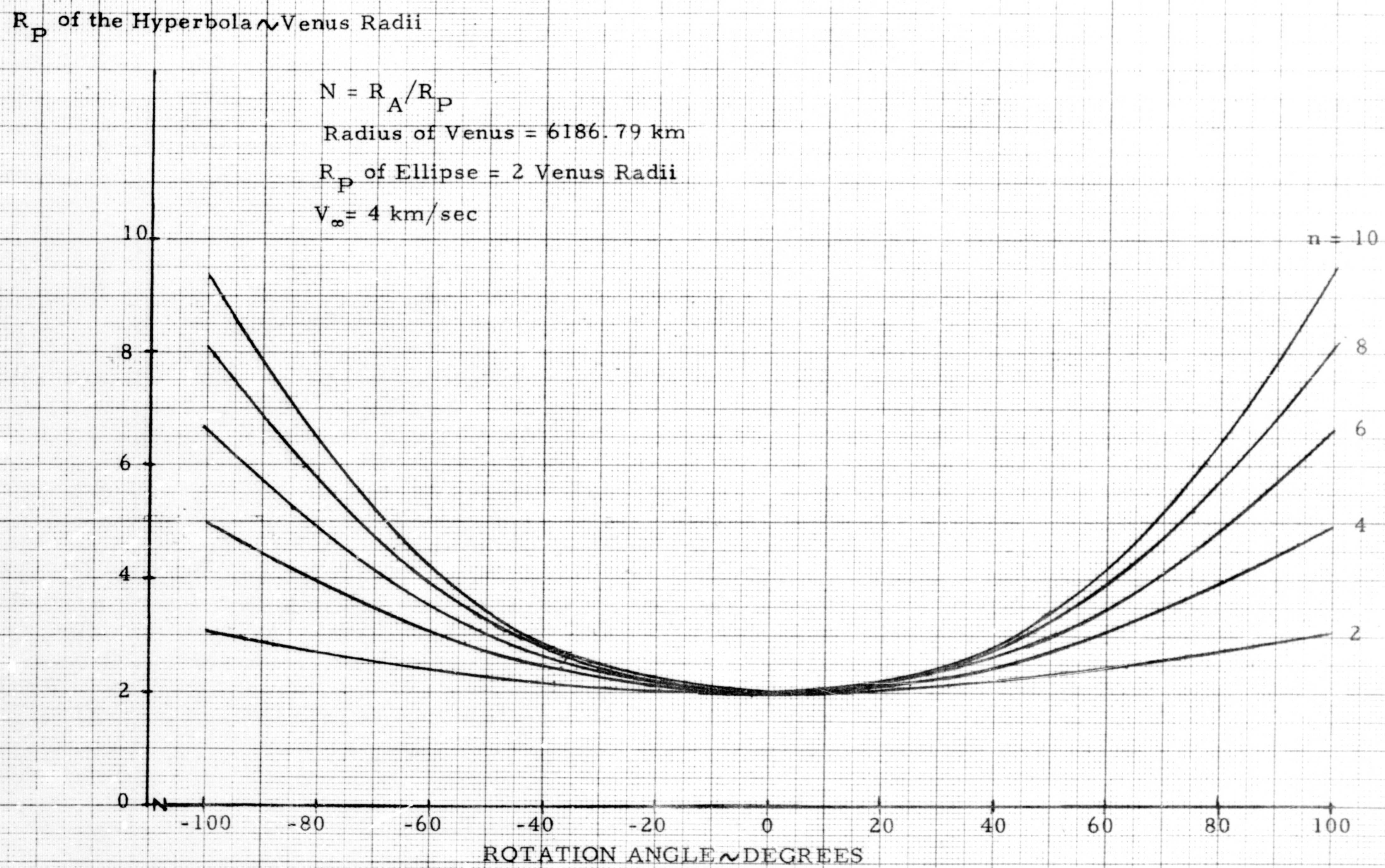
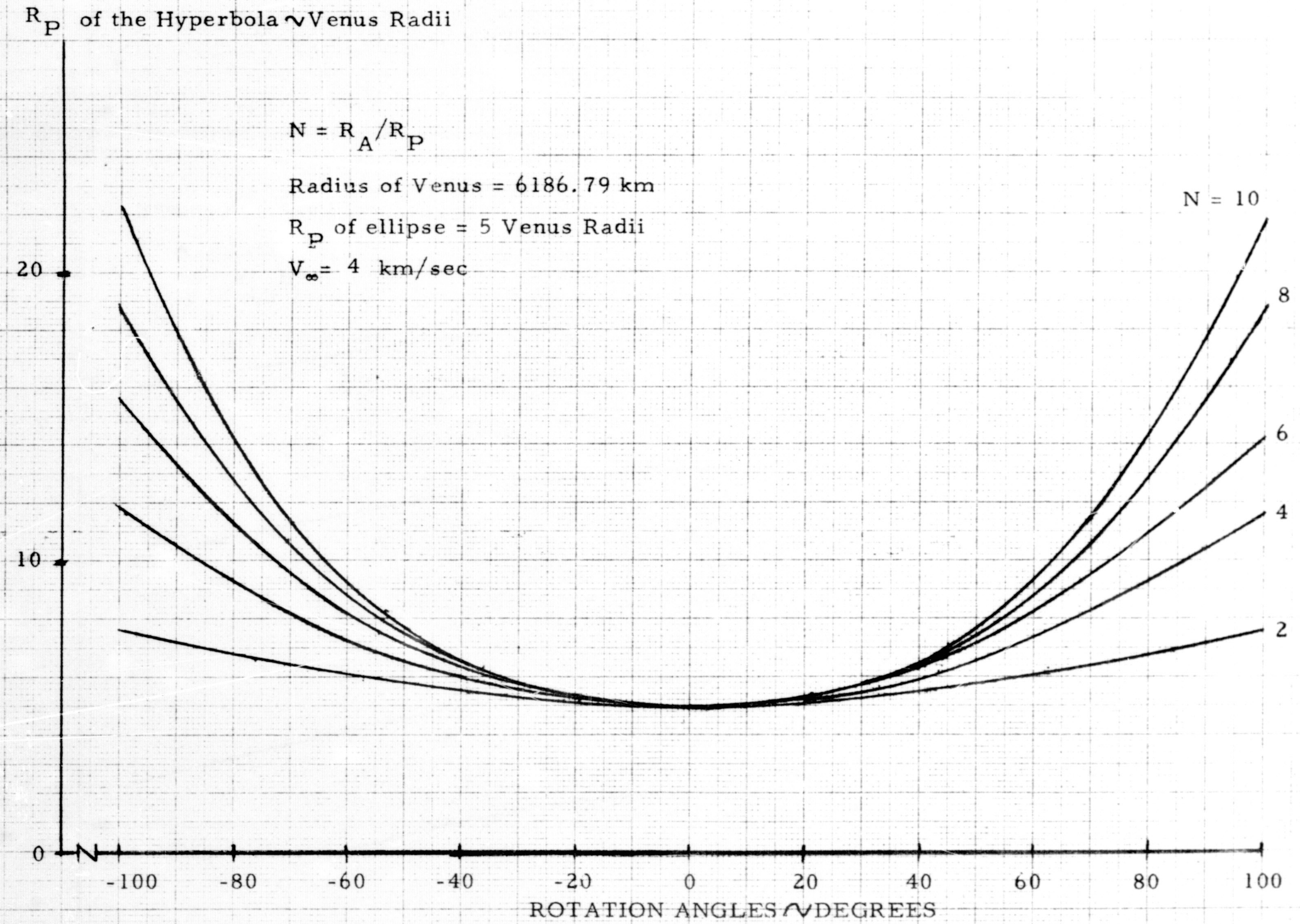


FIGURE 34 PERIAPSIS OF THE INCOMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO VENUS ORBIT



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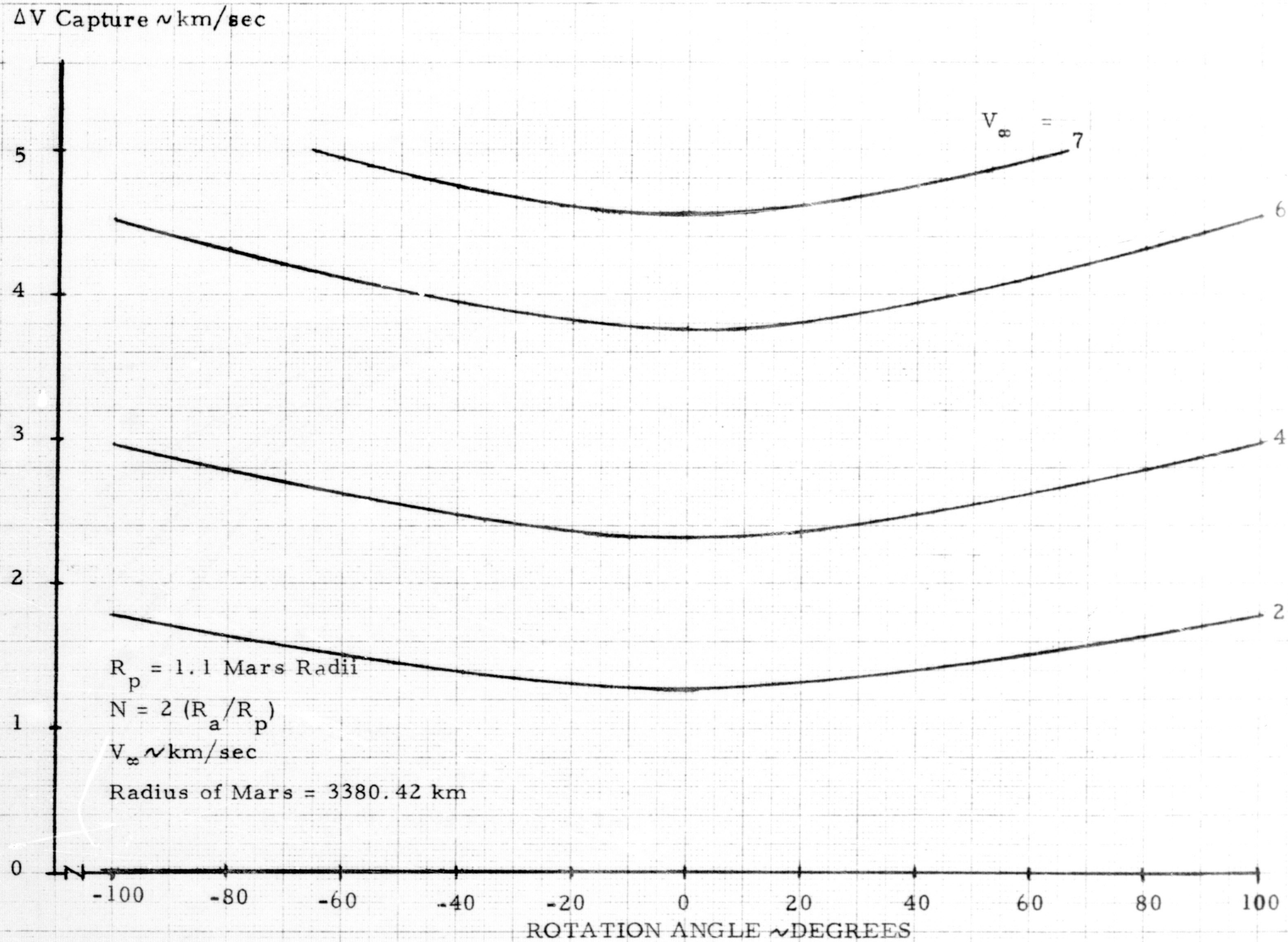


FIGURE 36 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

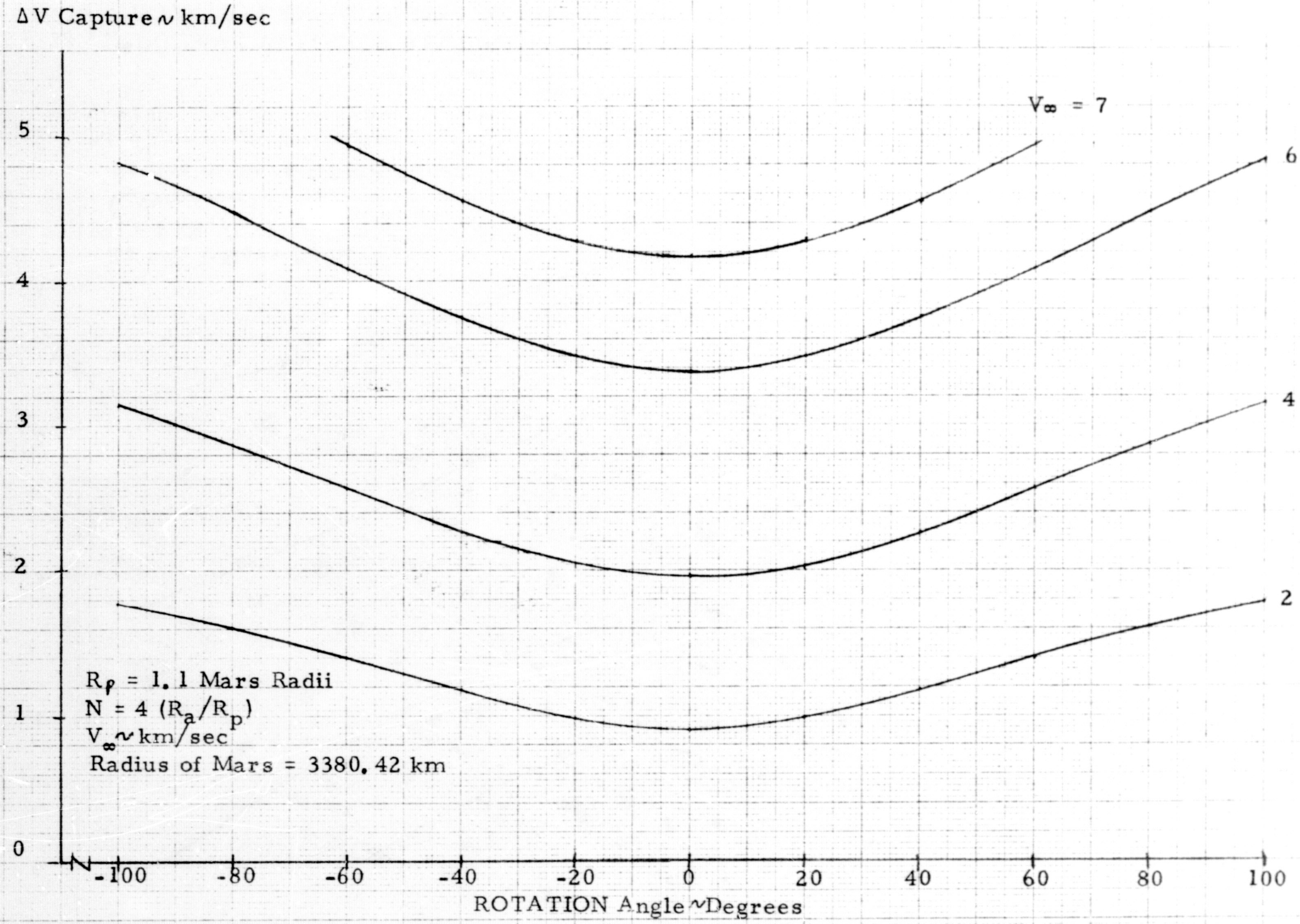


FIGURE 37 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

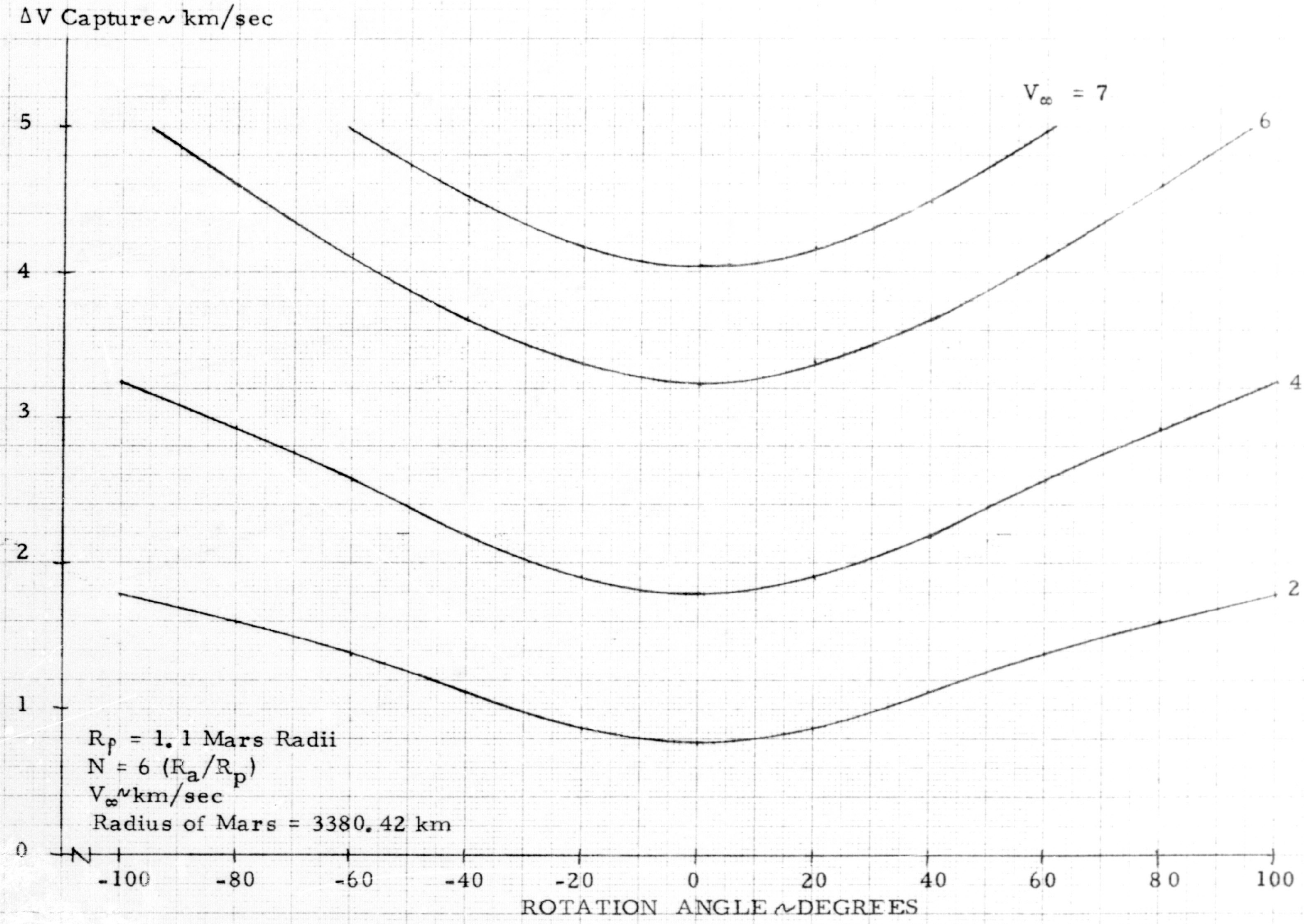


FIGURE 38 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

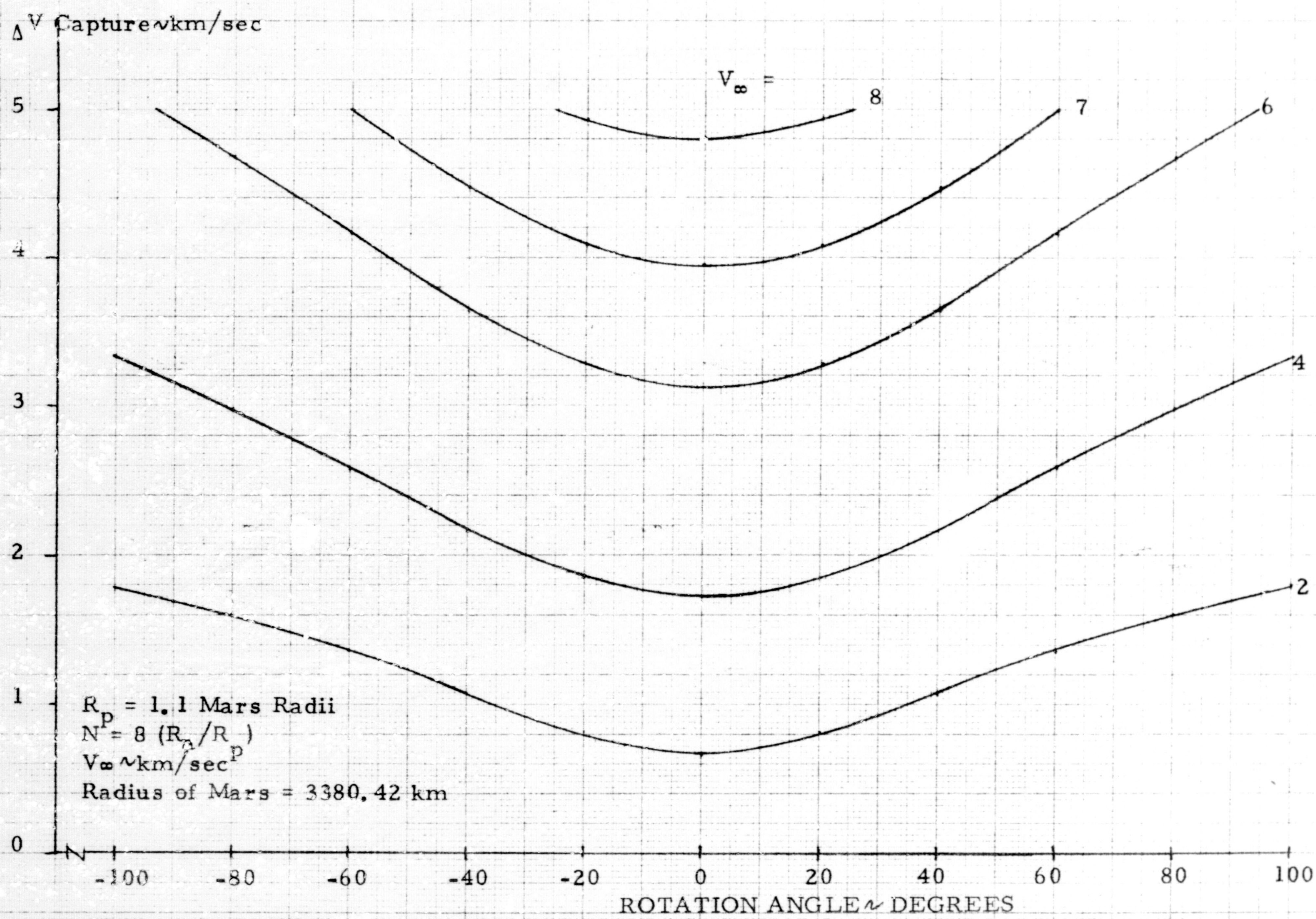


FIGURE 39 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

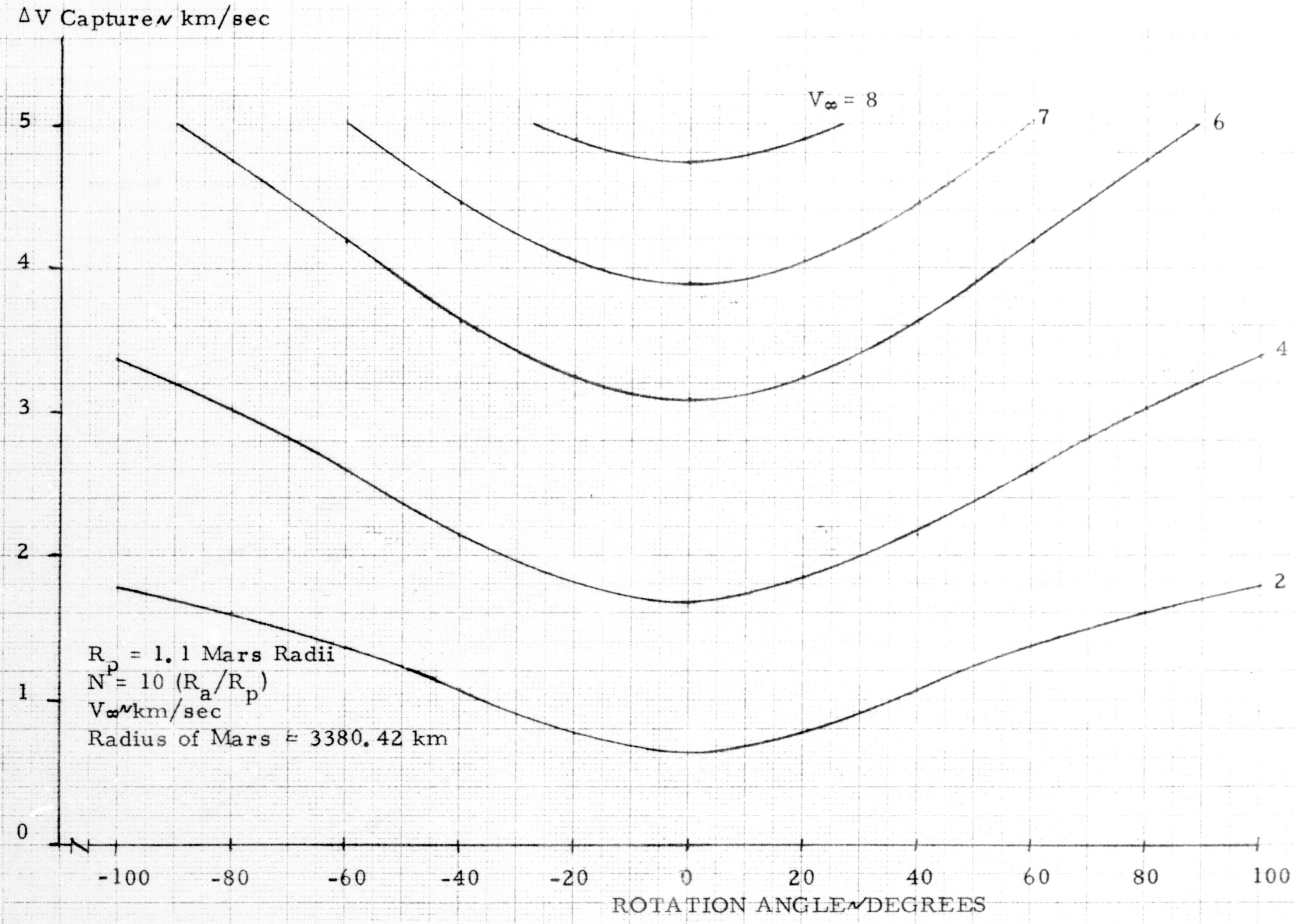


FIGURE 40 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

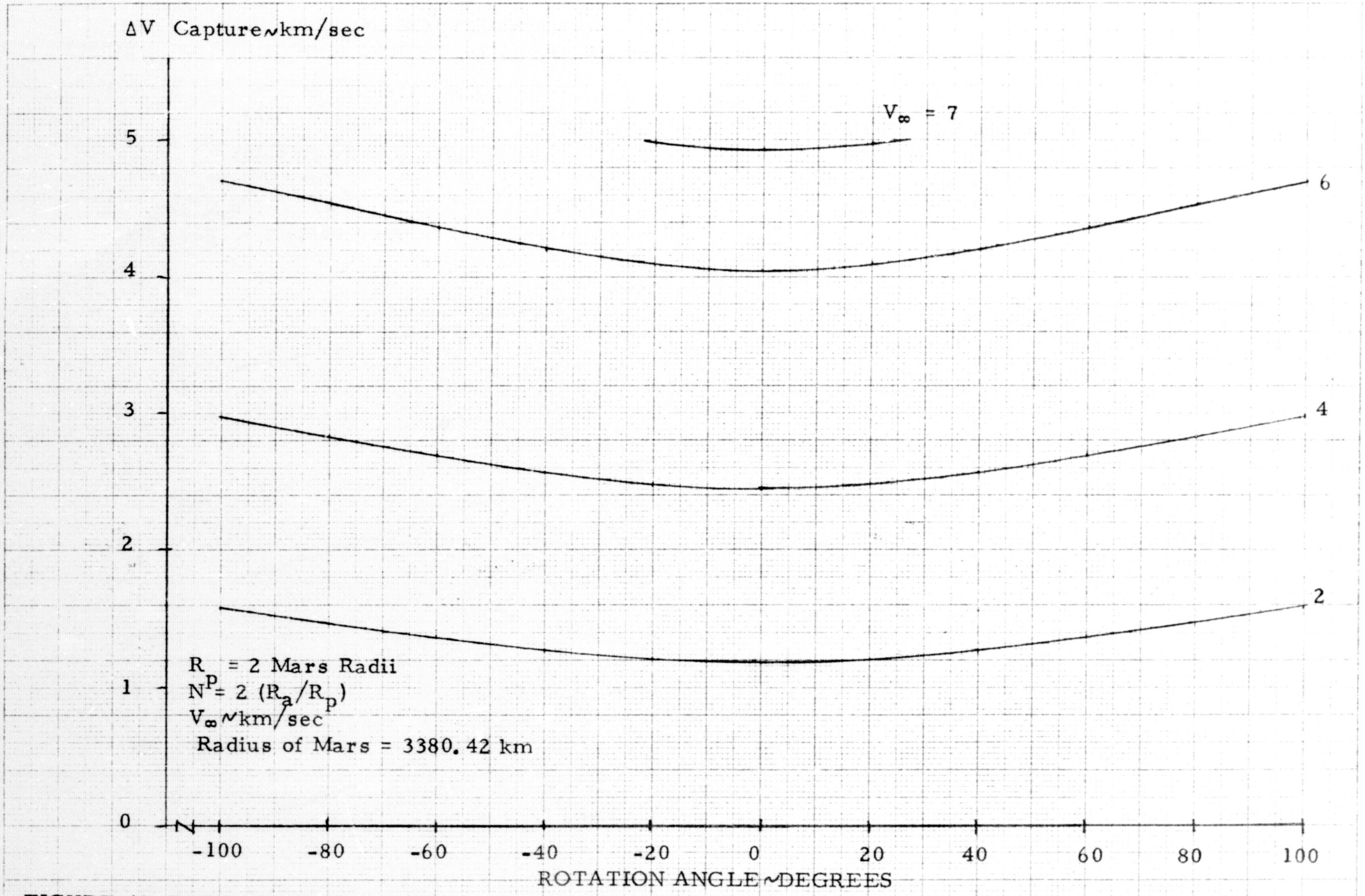


FIGURE 41 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

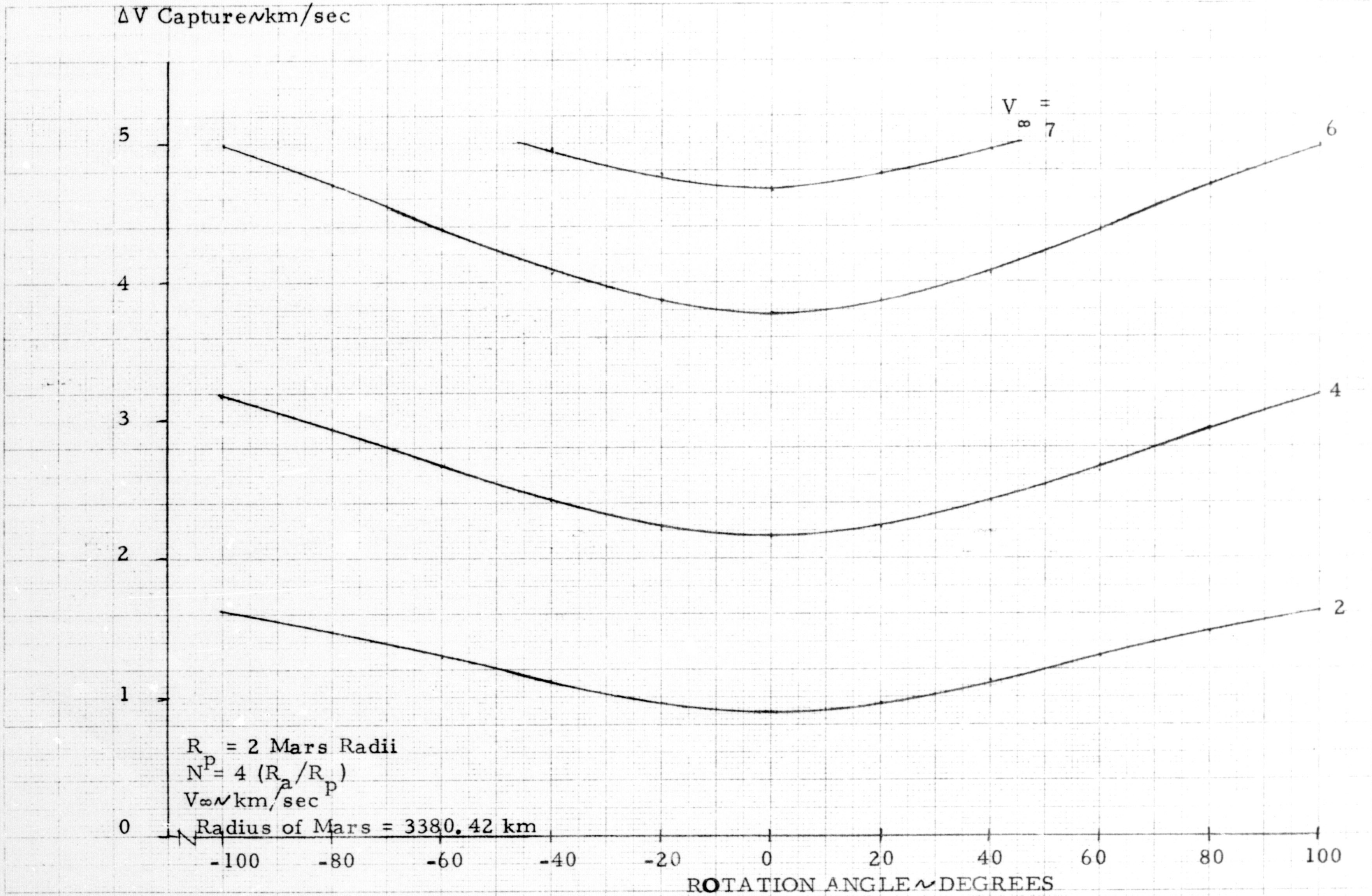


FIGURE 42 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

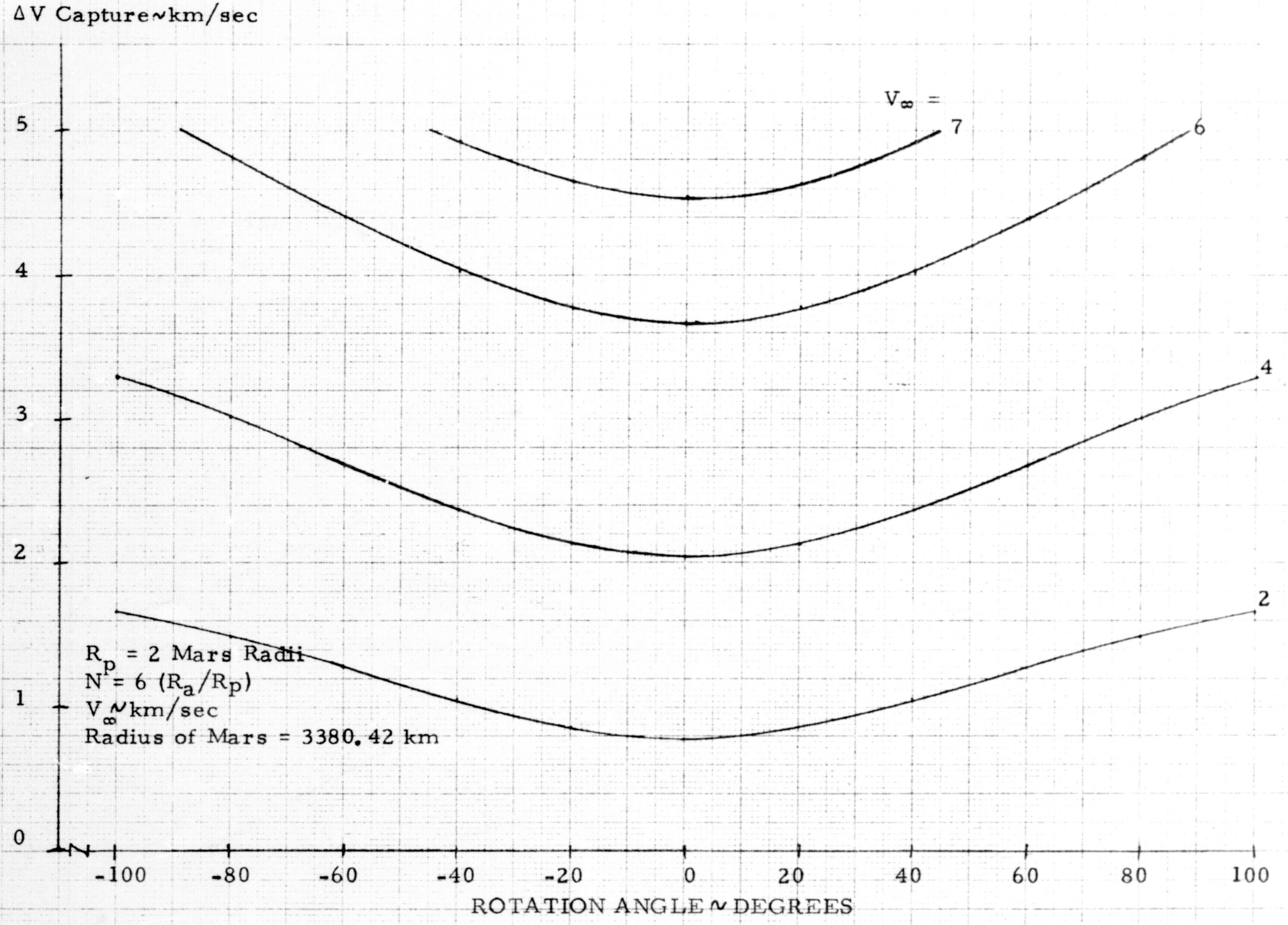


FIGURE 43 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

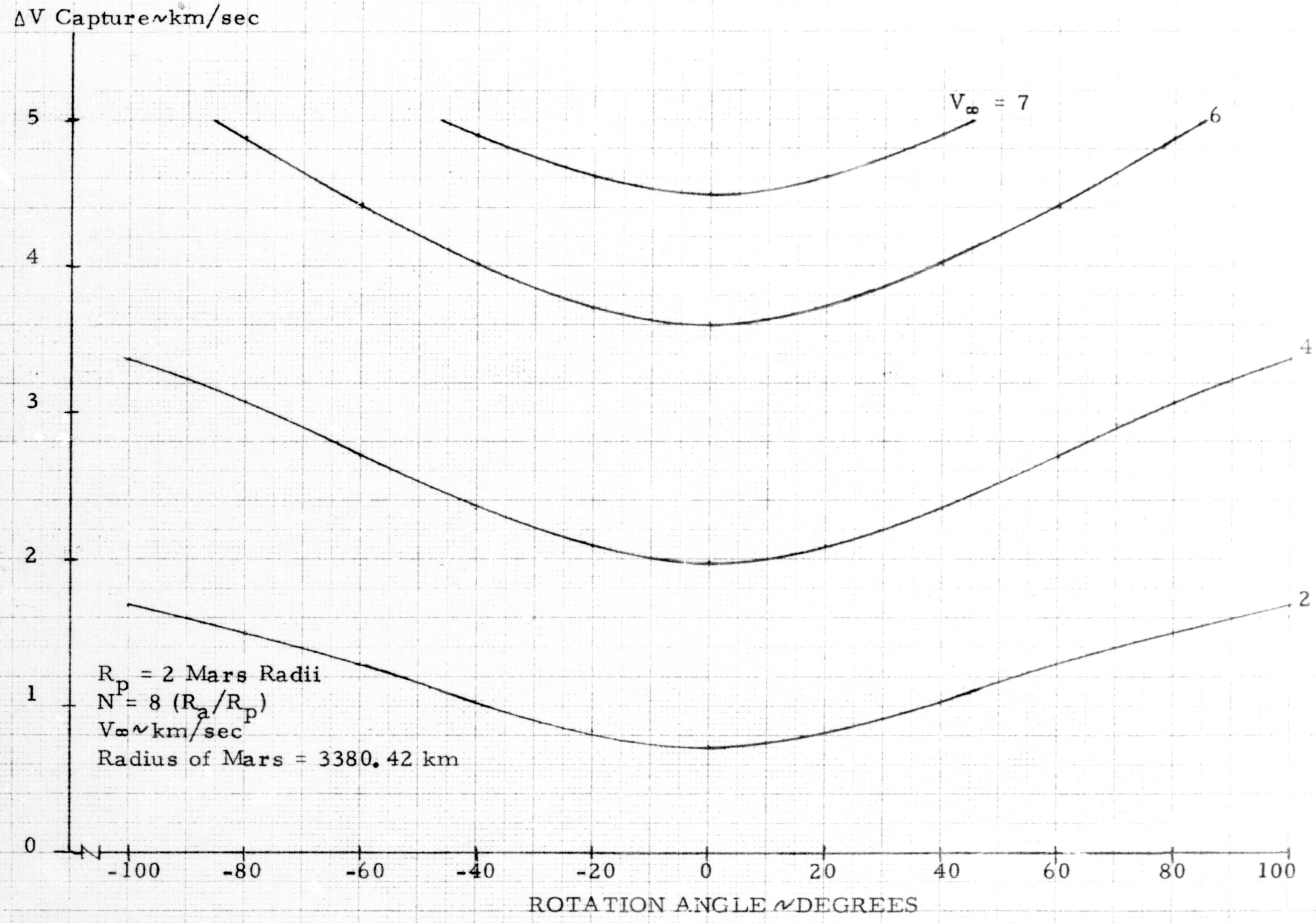


FIGURE 44 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

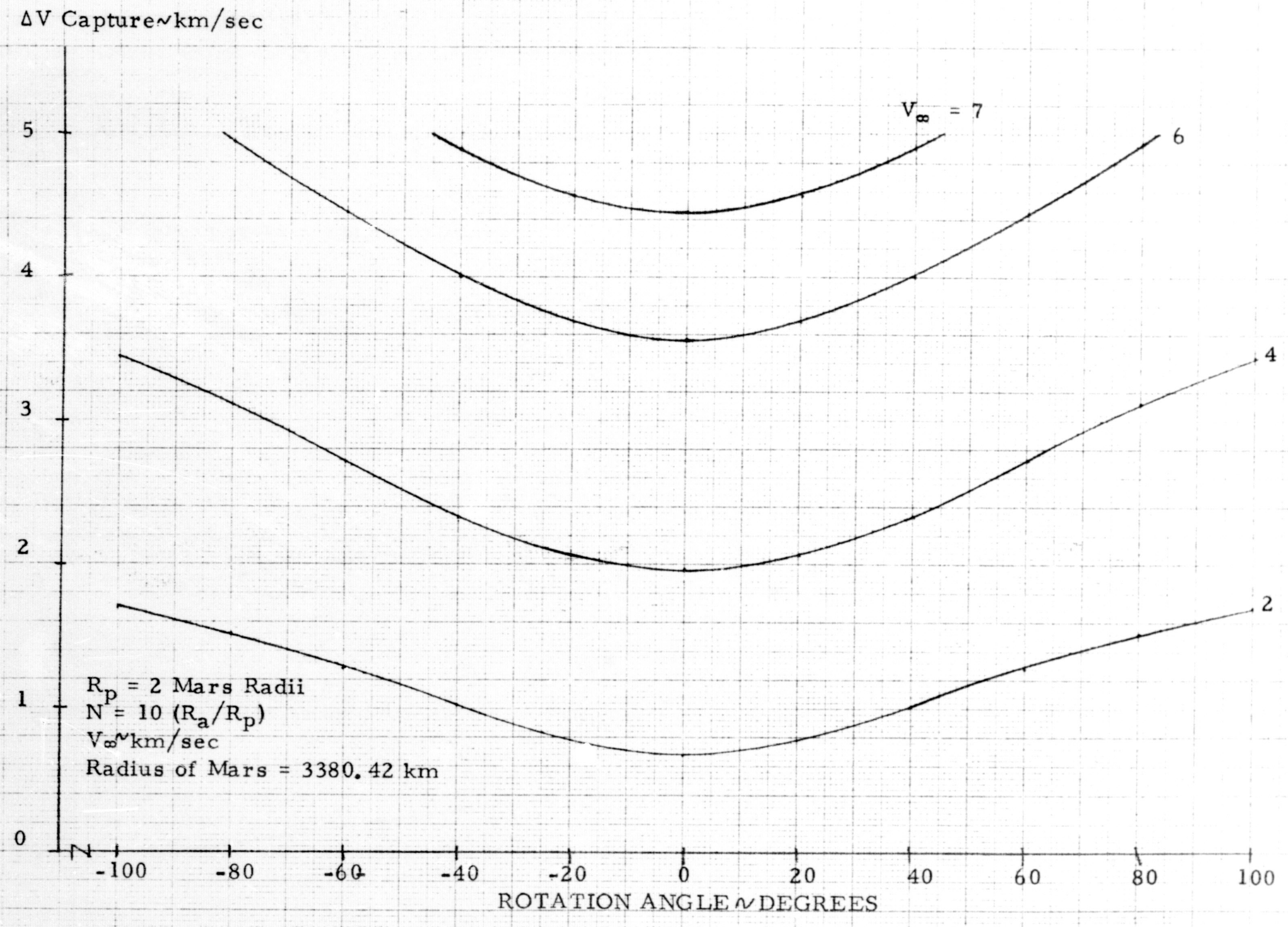


FIGURE 45 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

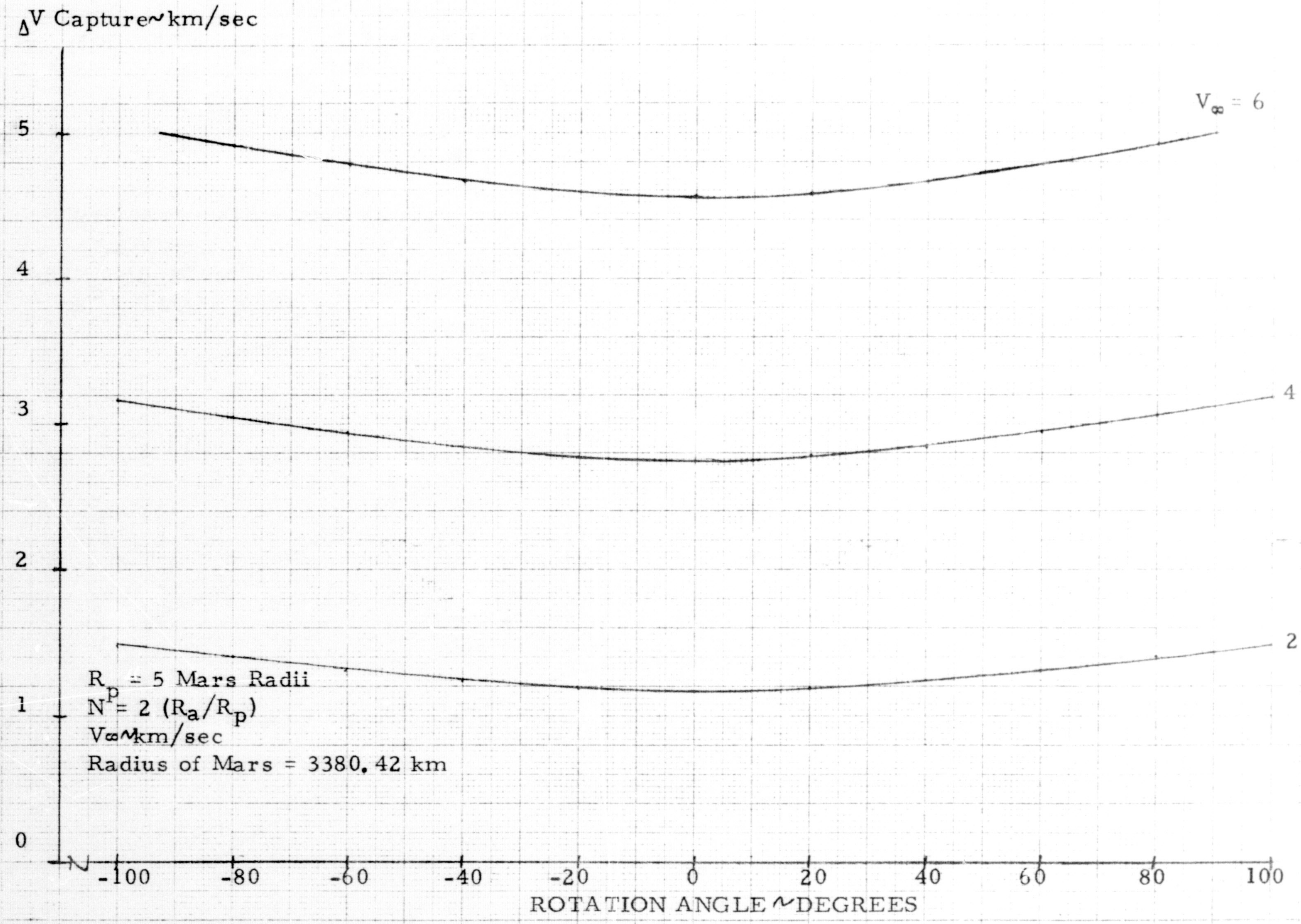


FIGURE 46 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

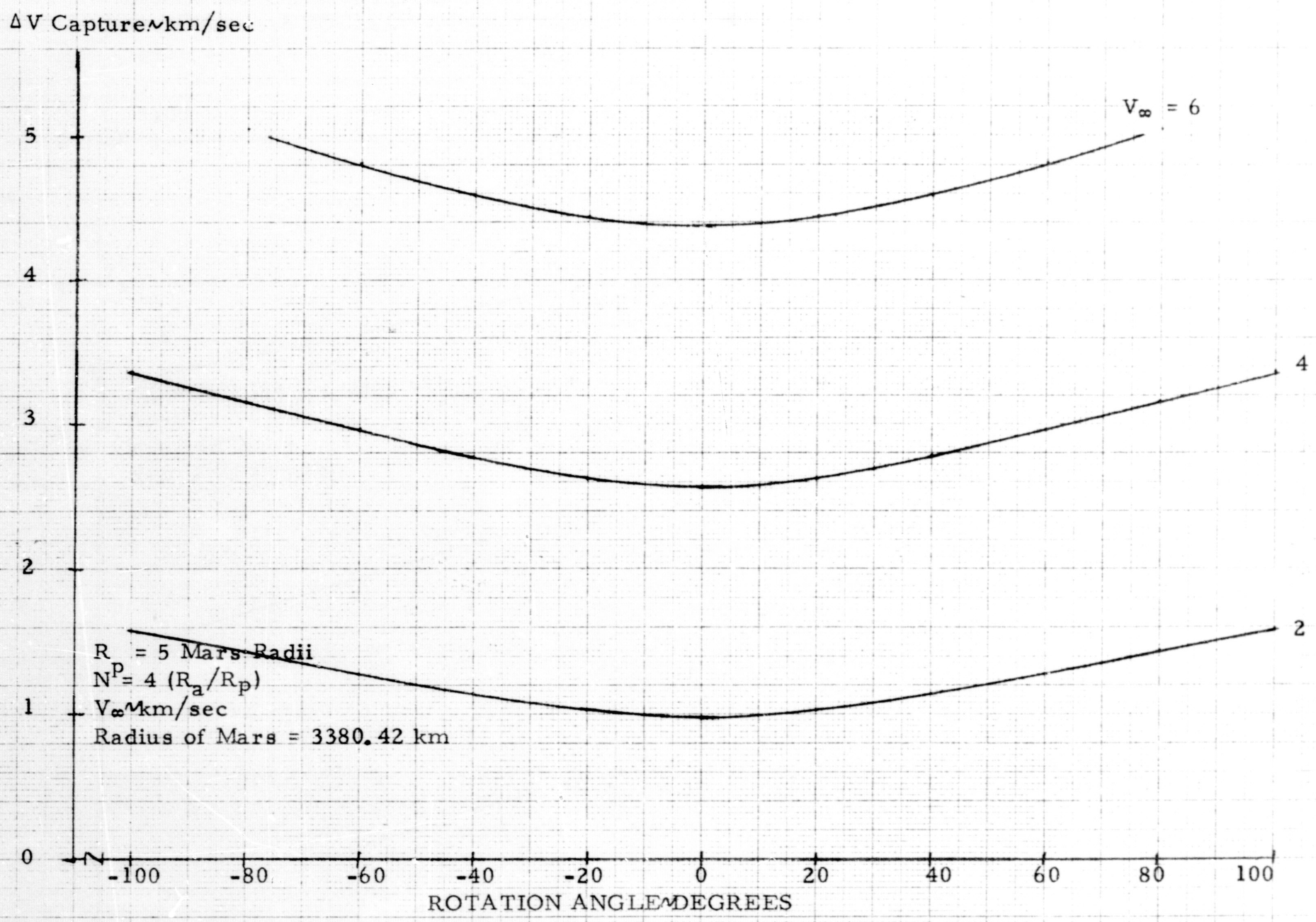
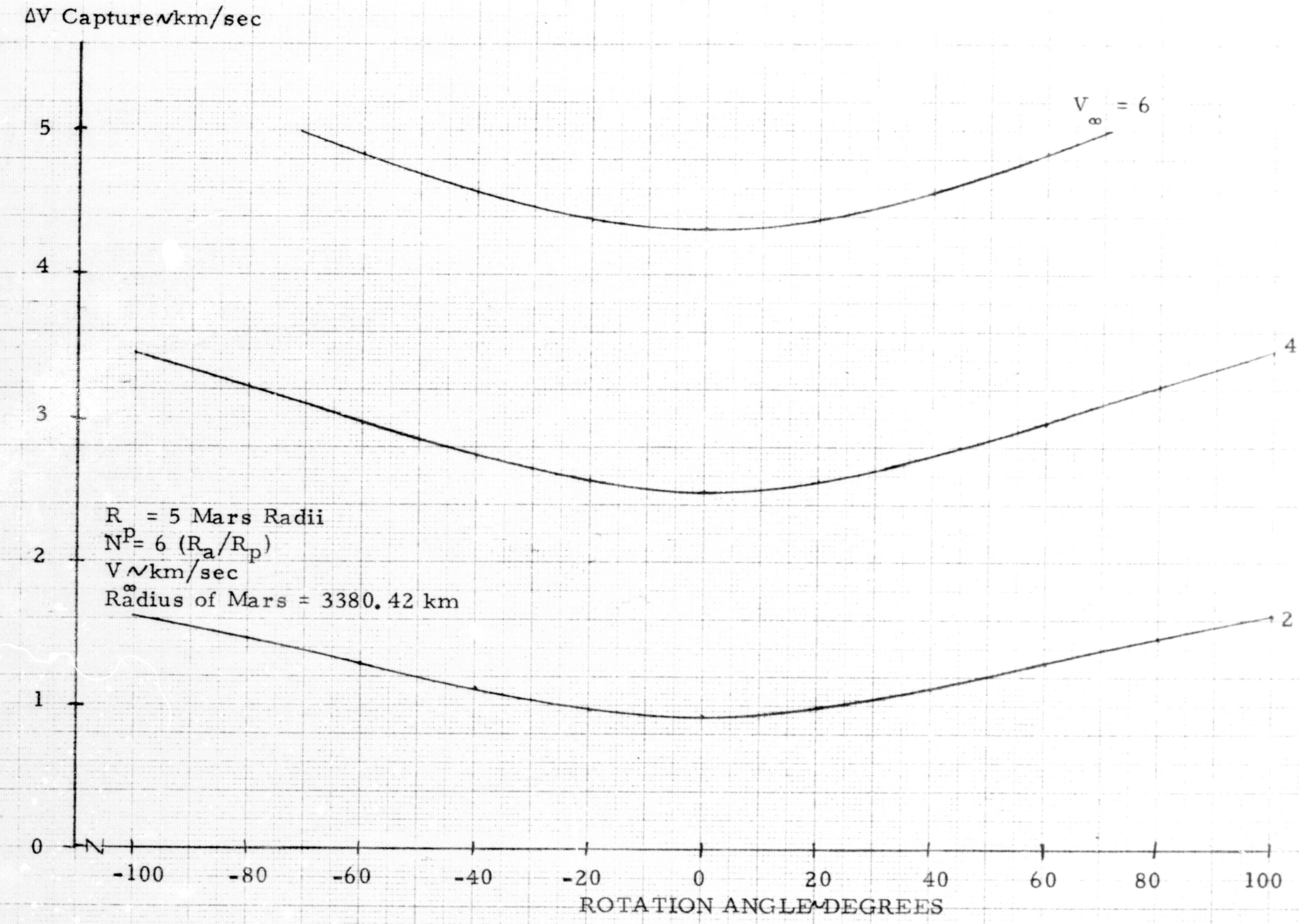


FIGURE 47 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER



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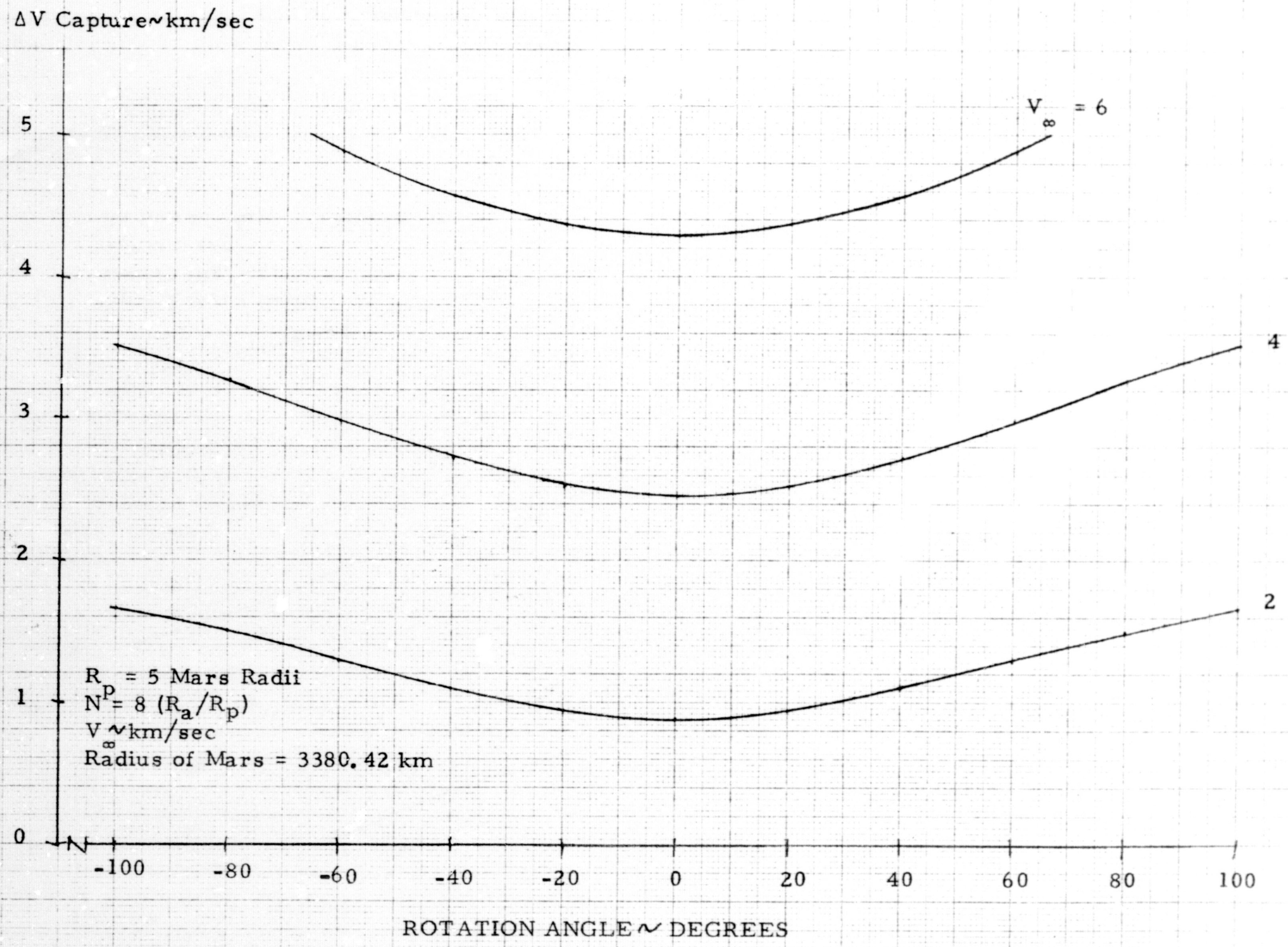


FIGURE 49 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

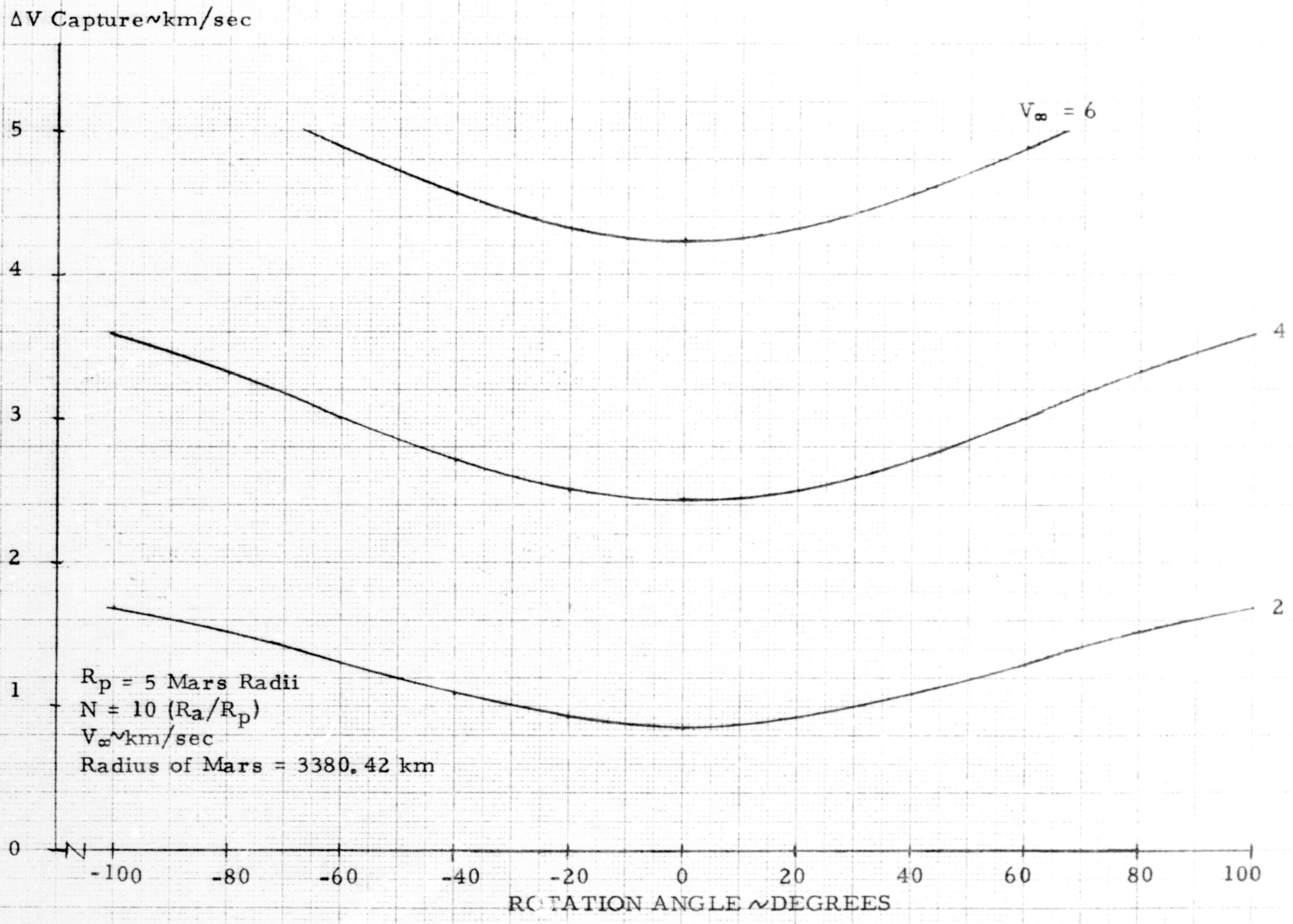


FIGURE 50 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

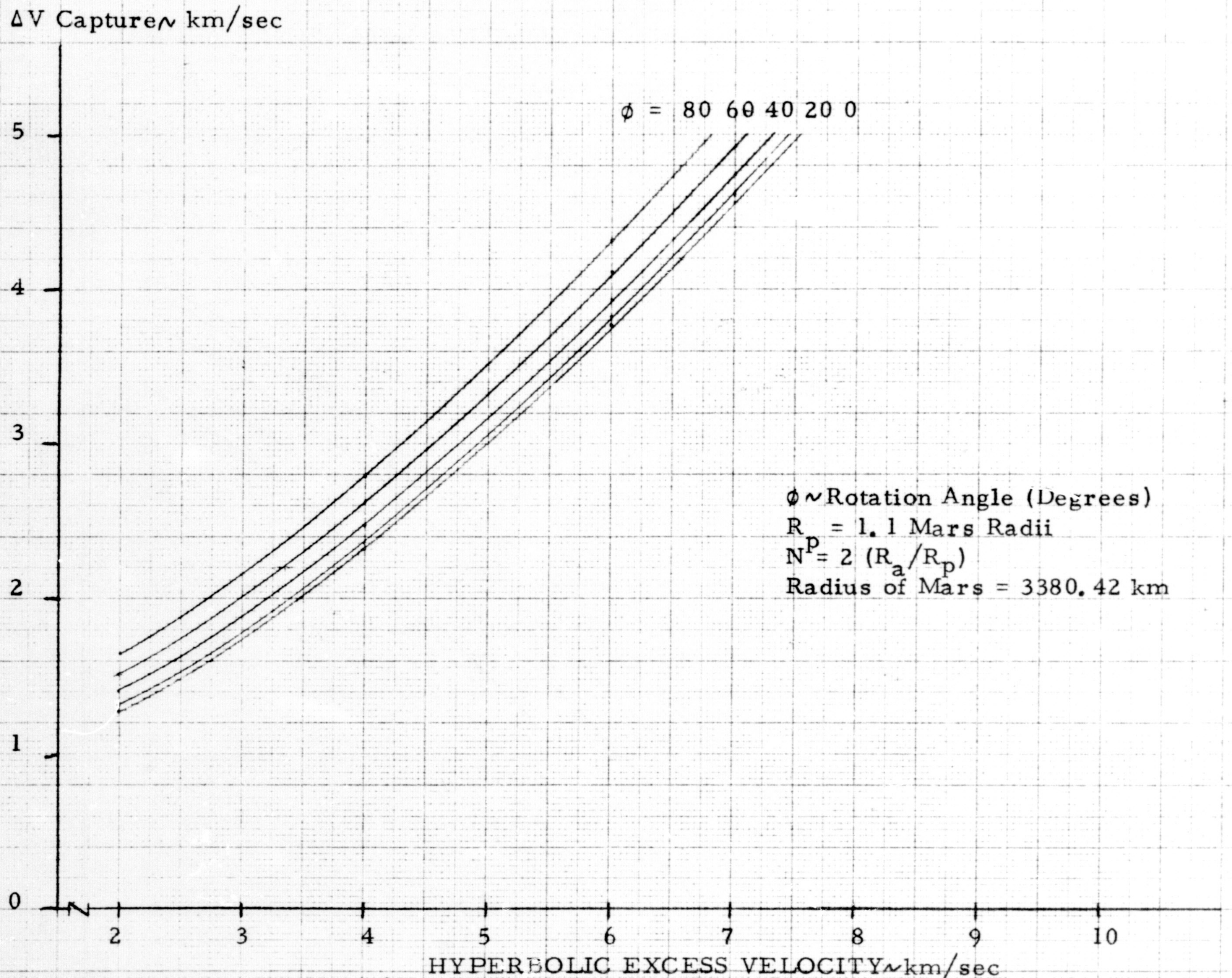


FIGURE 51 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) for TANGENTIAL TRANSFER

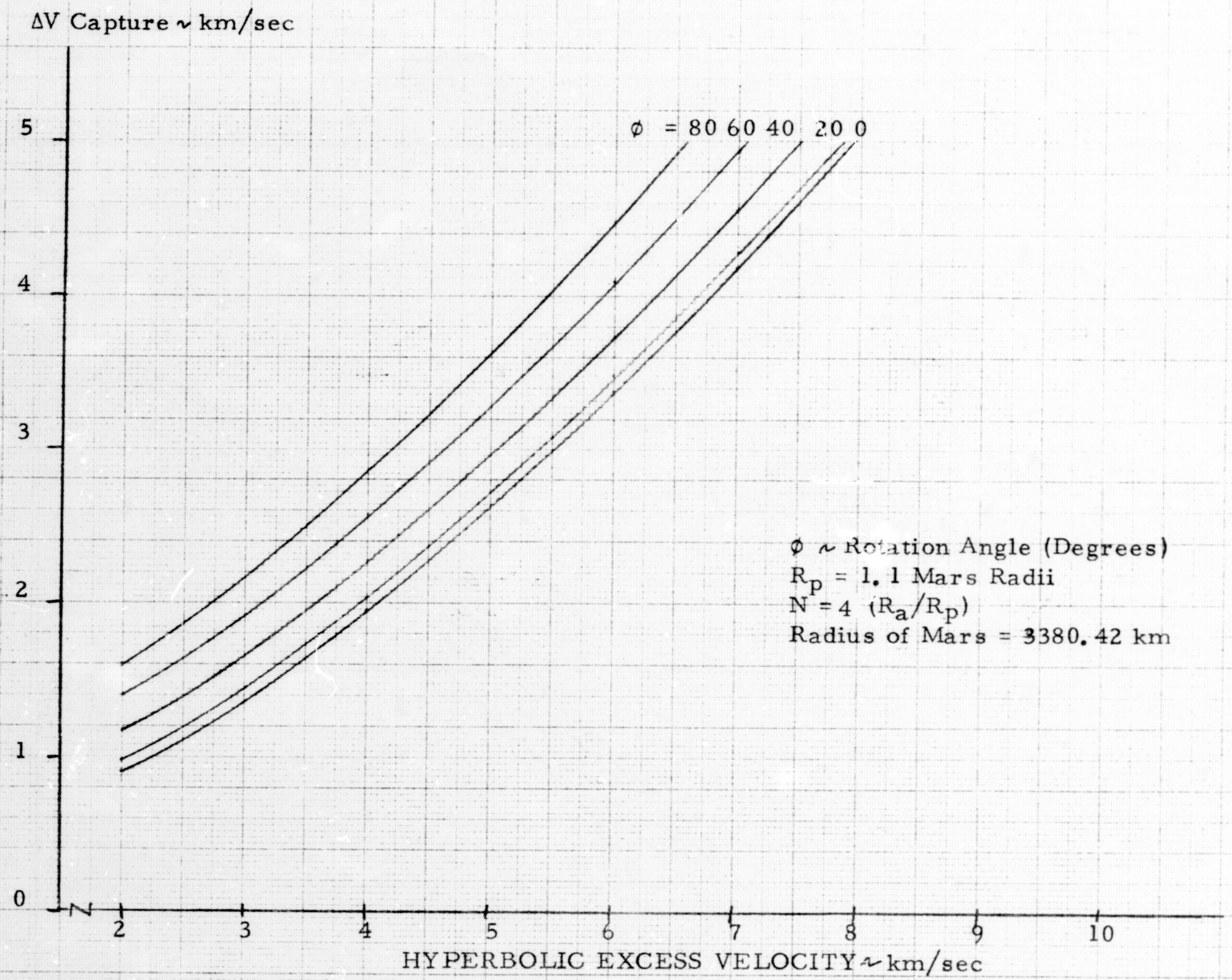


FIGURE 52 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

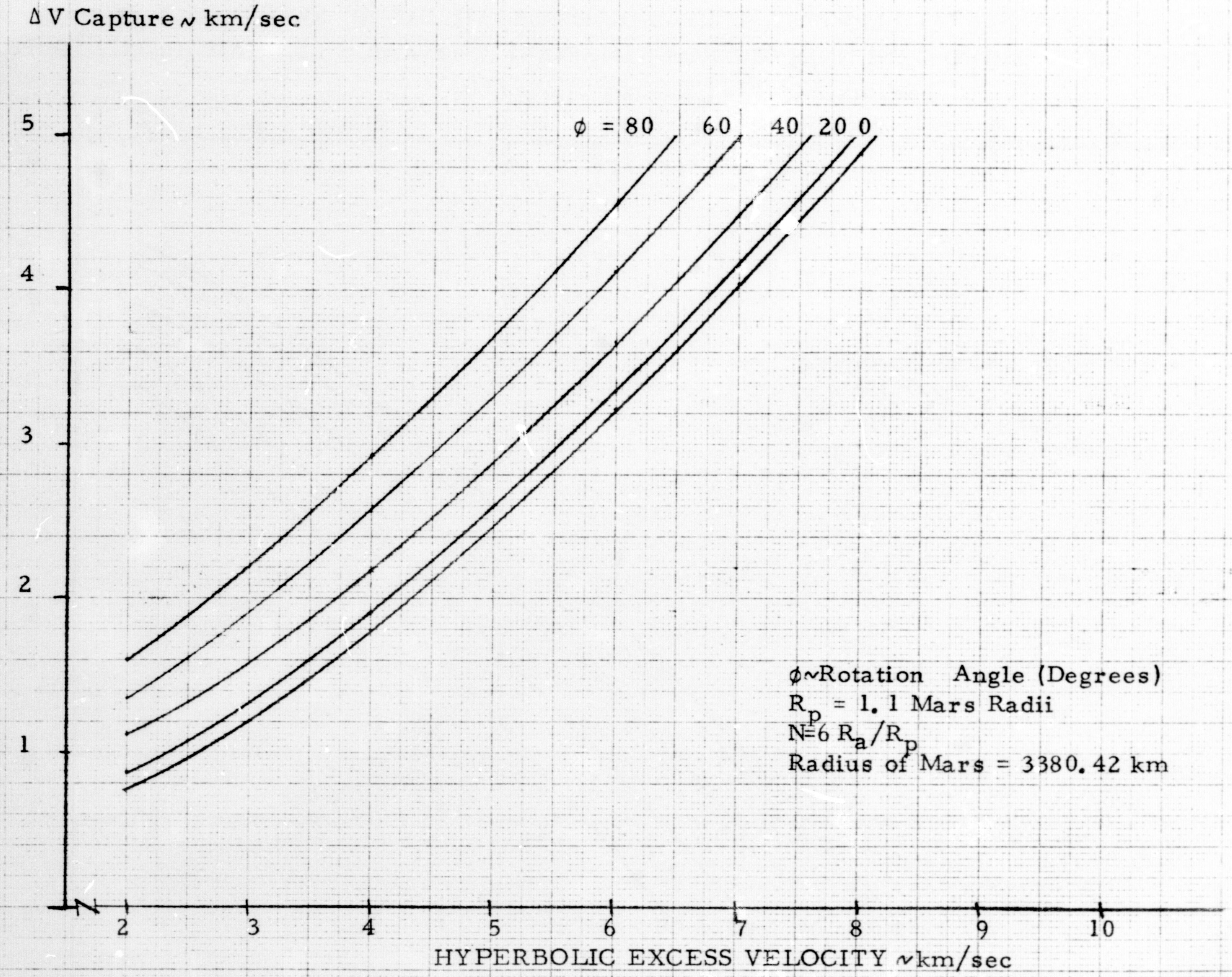


FIGURE 53 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

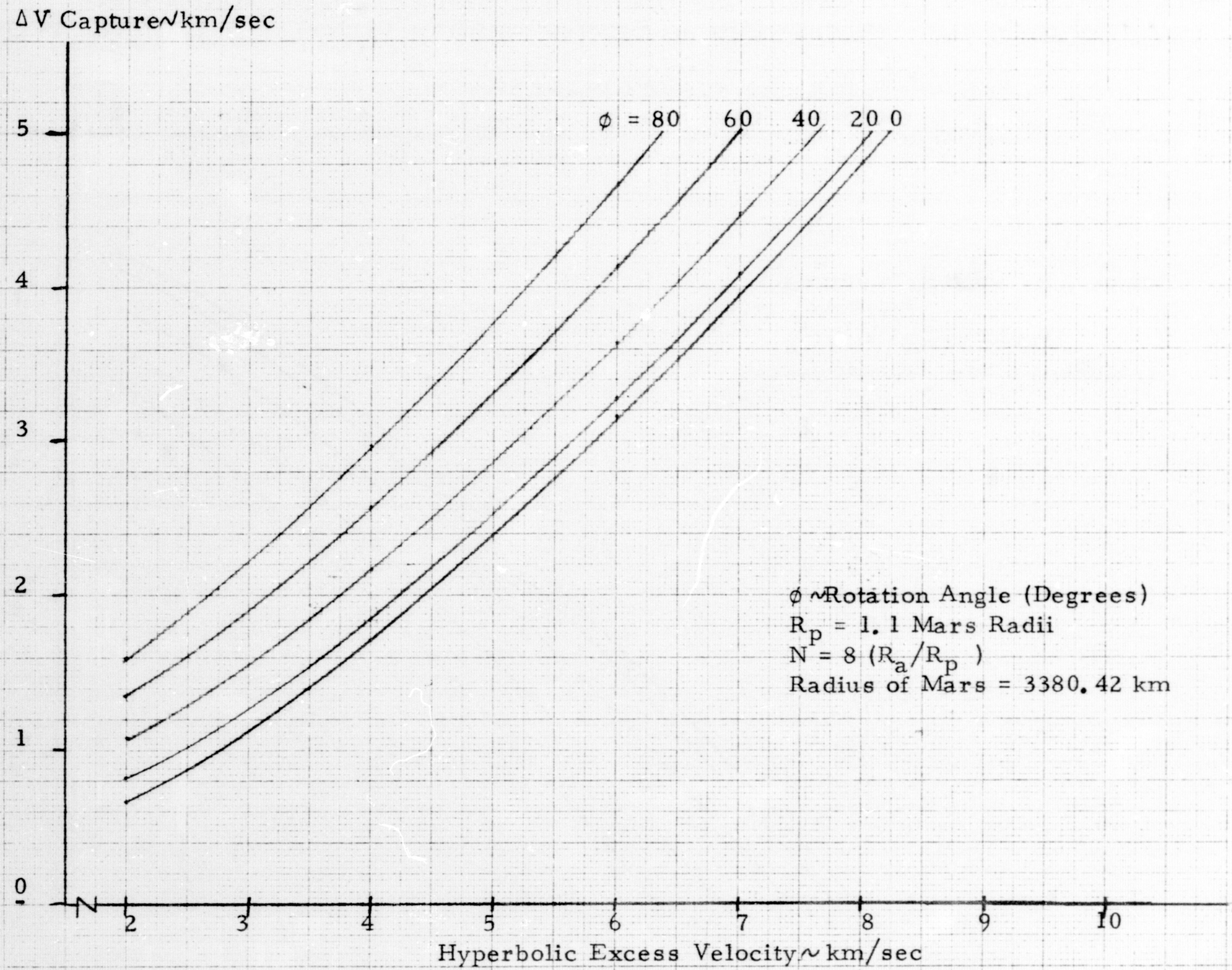


FIGURE 54 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

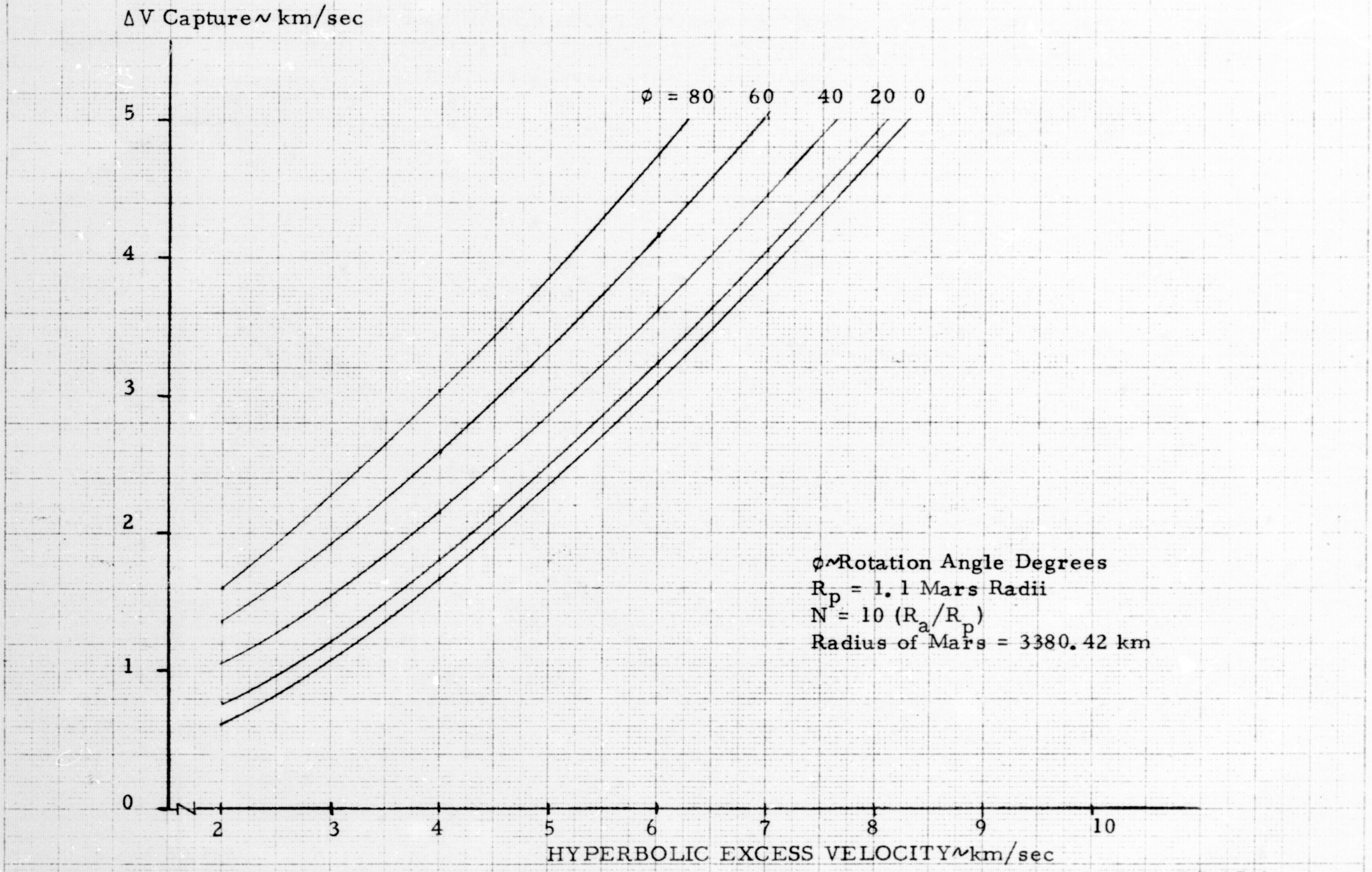


FIGURE 55 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

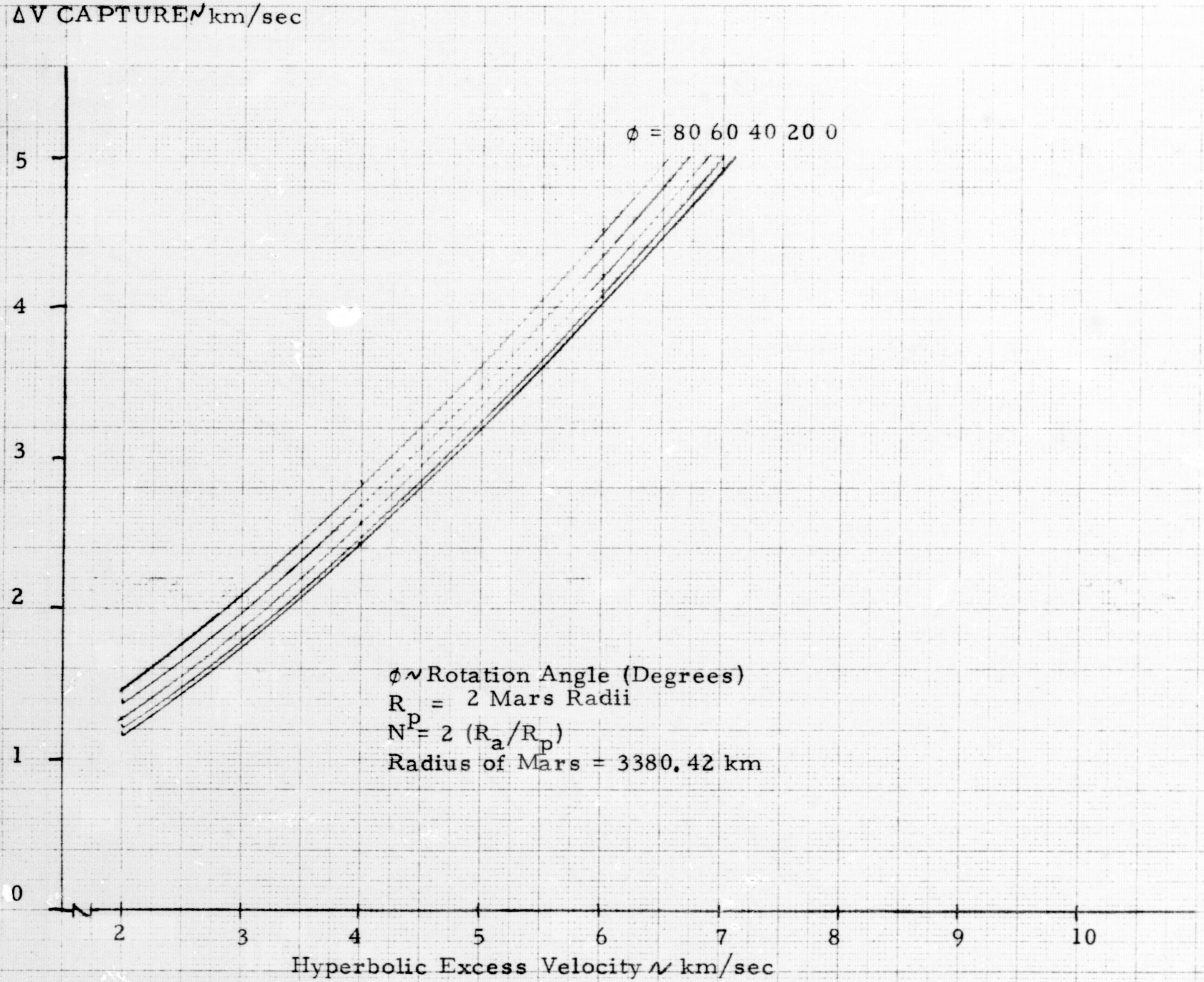
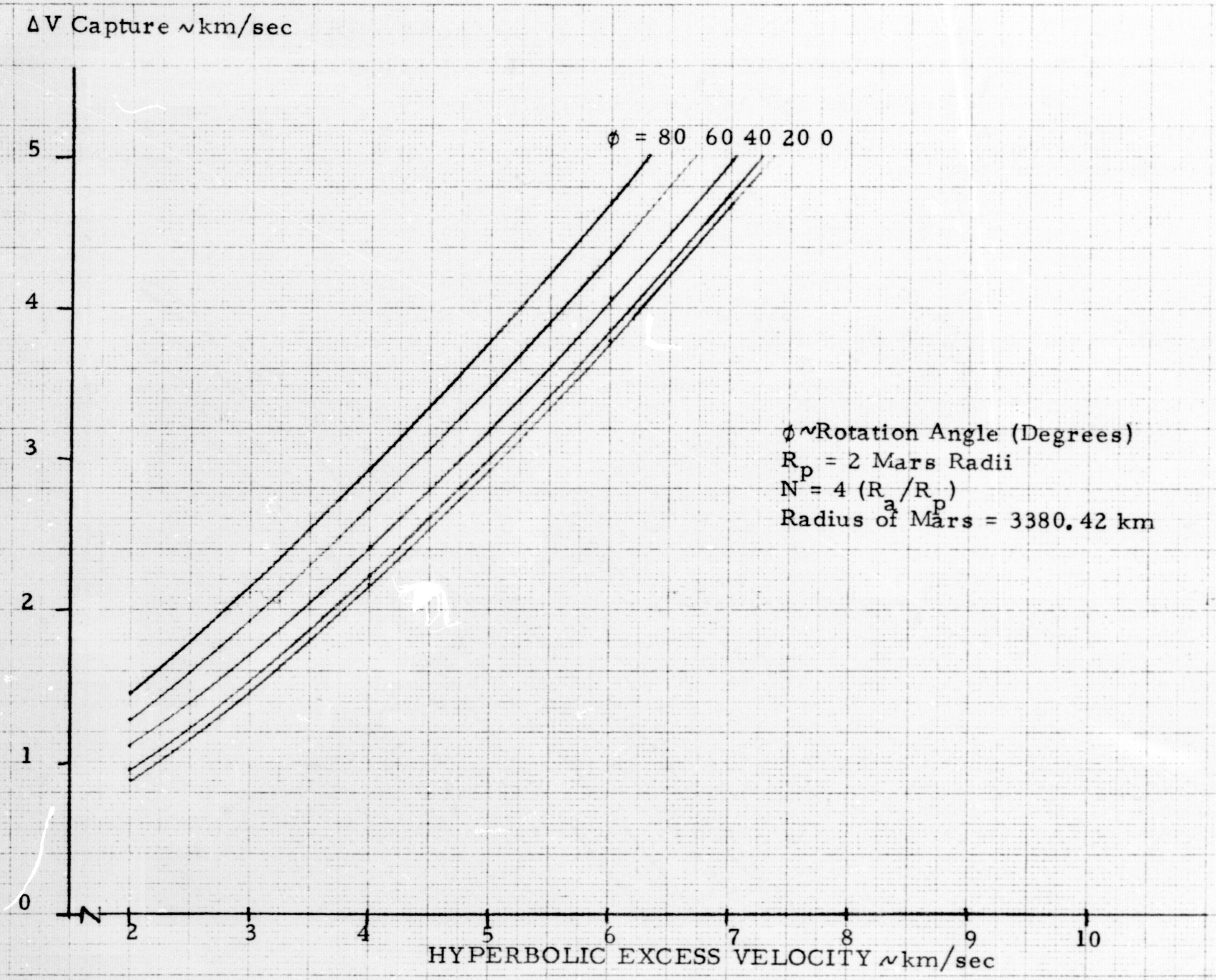


FIGURE 56 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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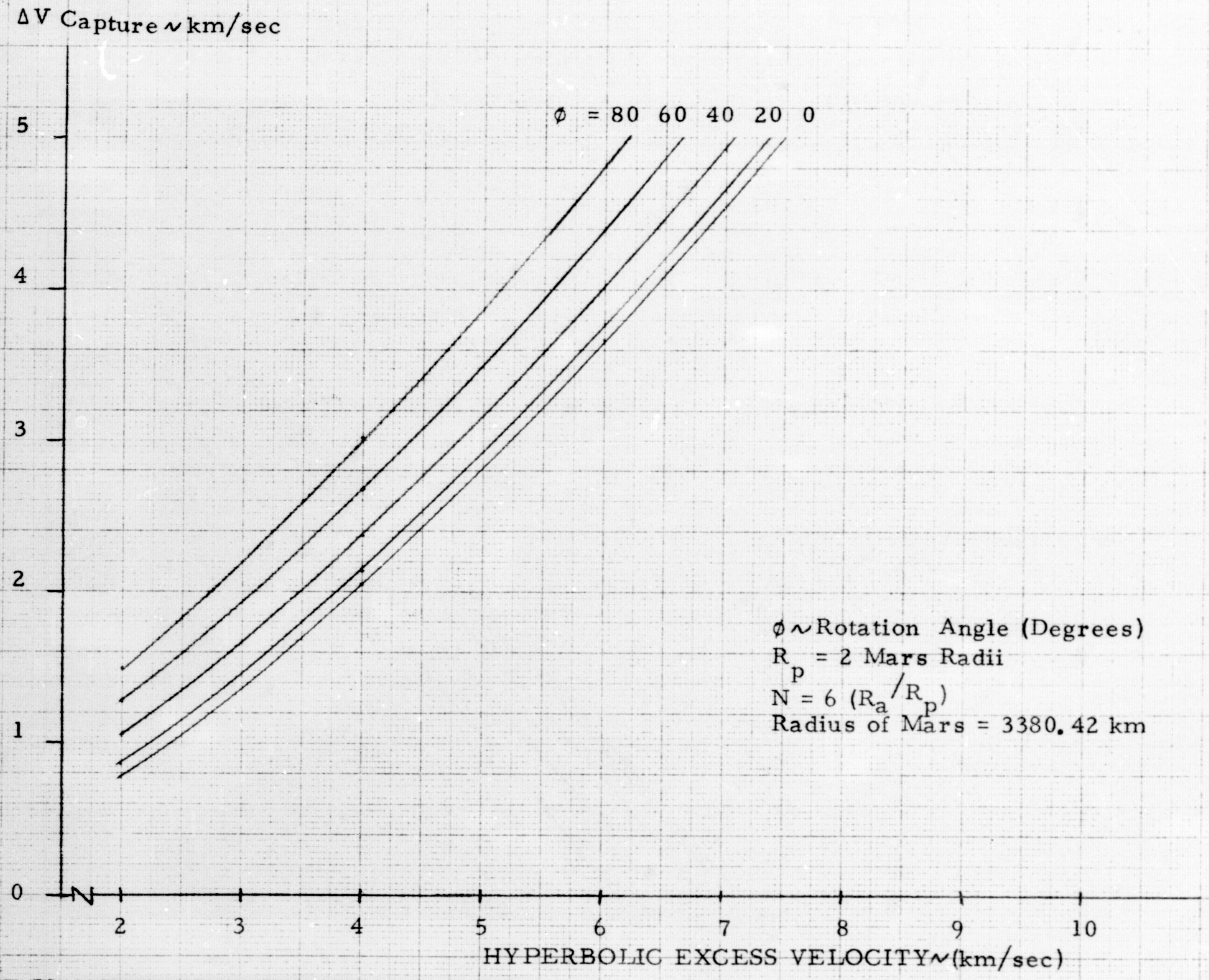
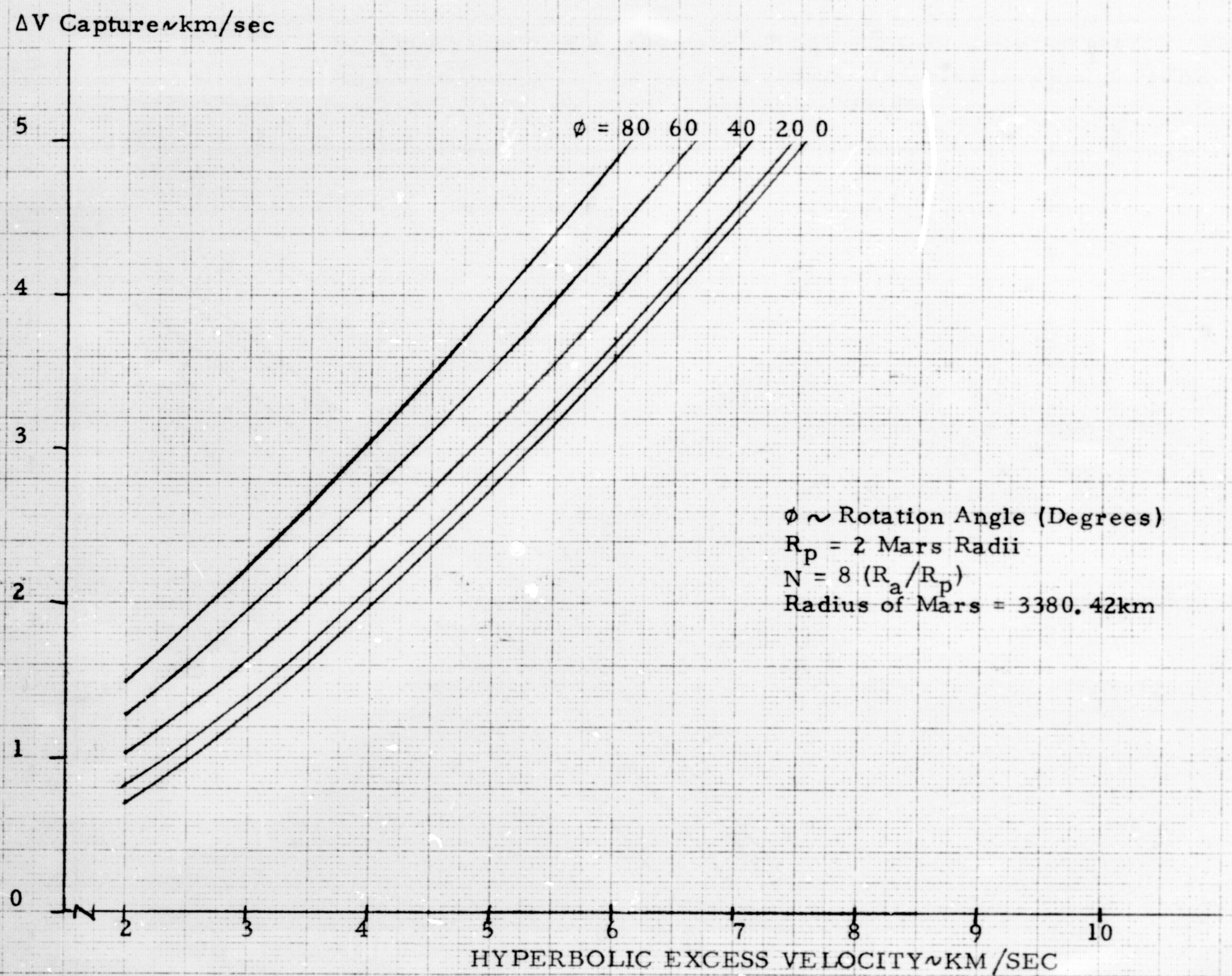


FIGURE 58 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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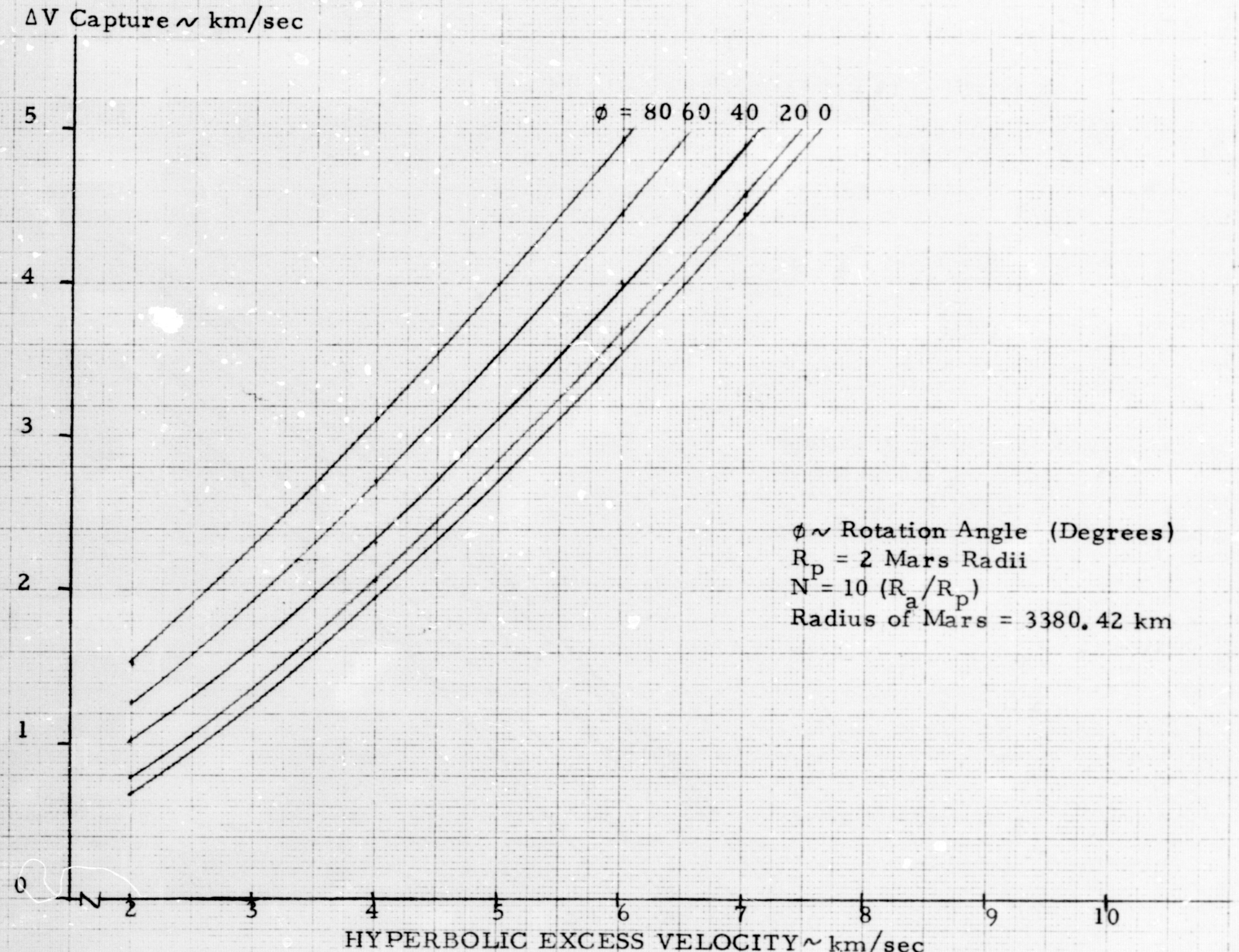


FIGURE 60 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

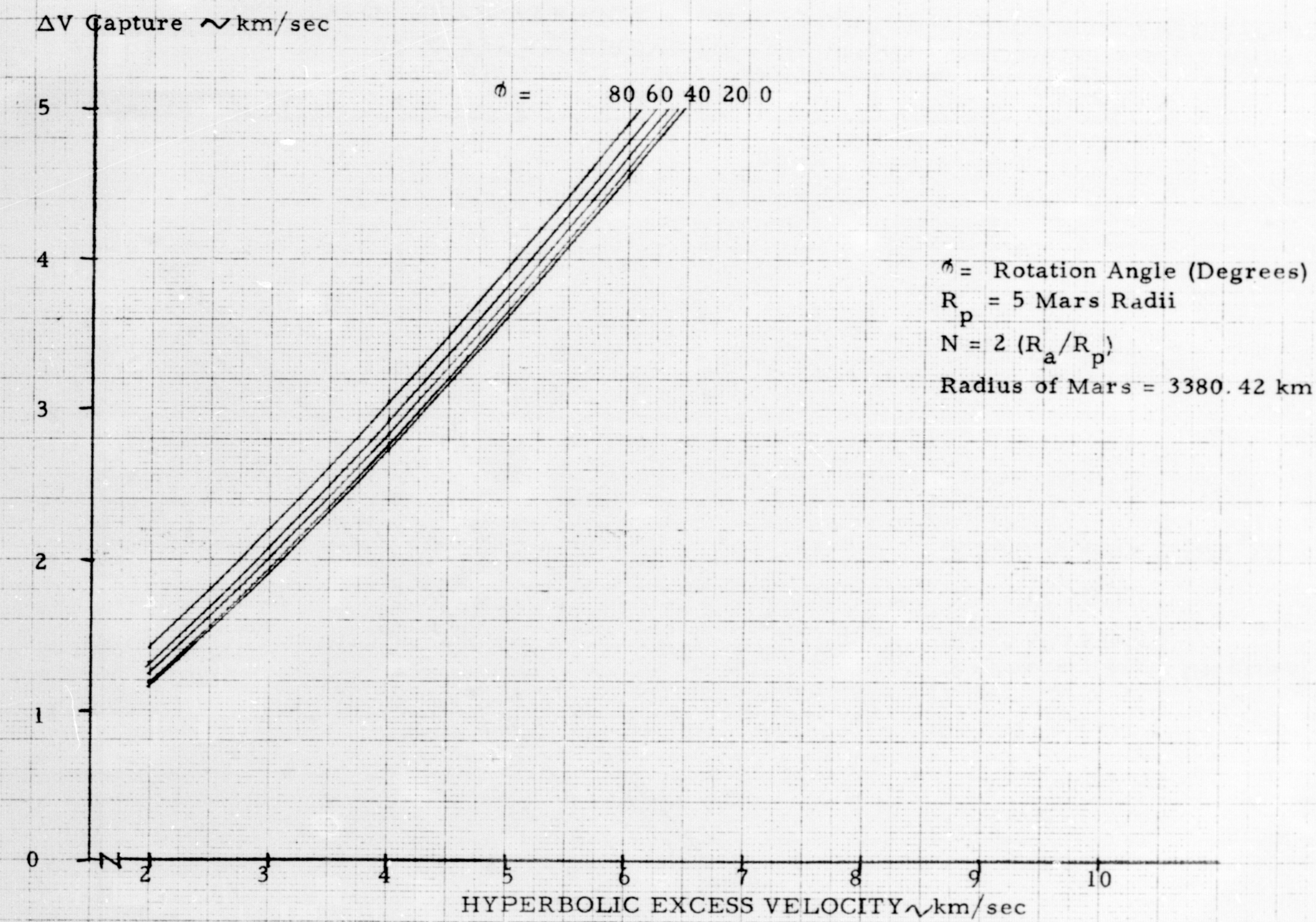
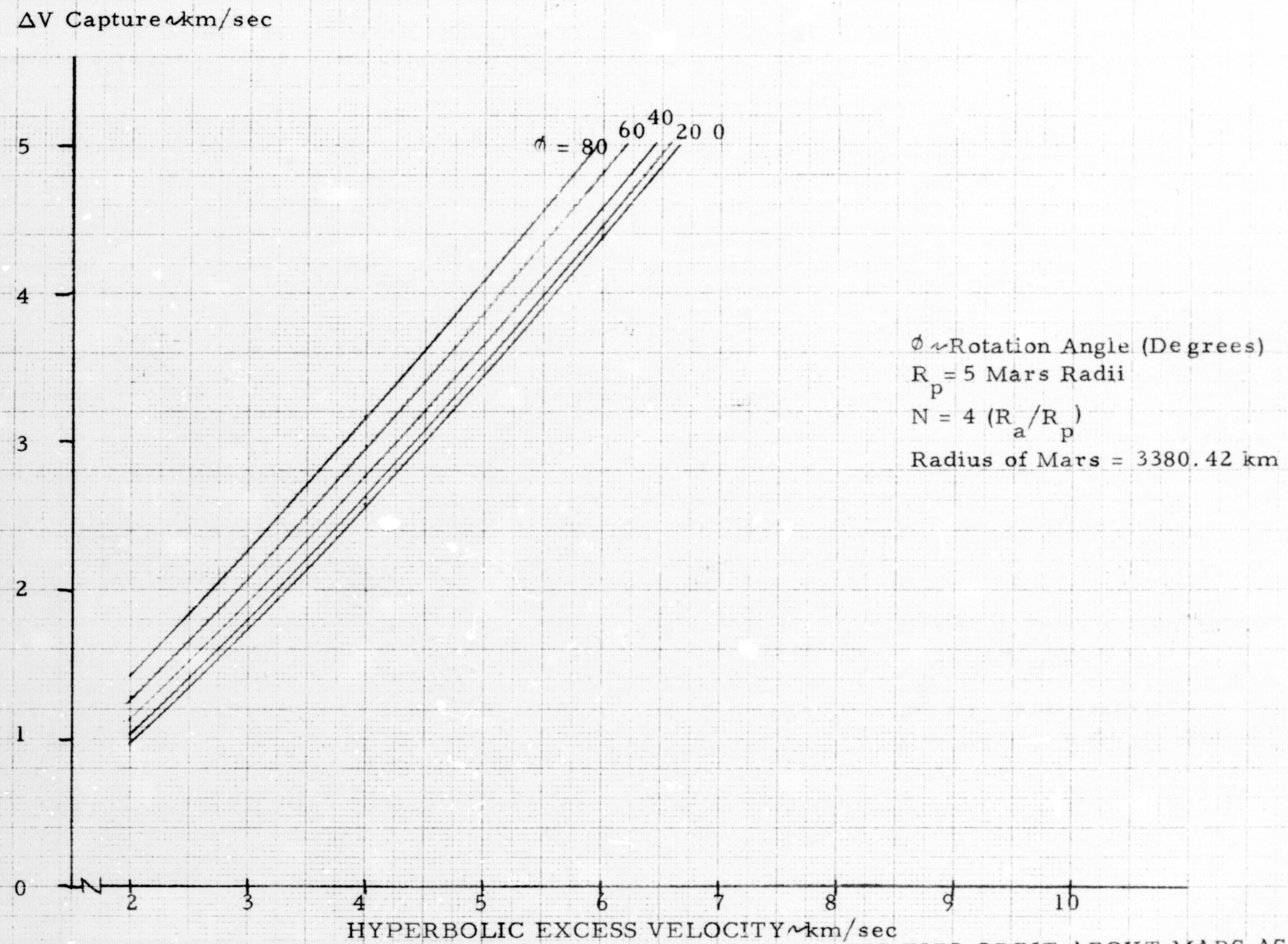


FIGURE 61 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT MARS AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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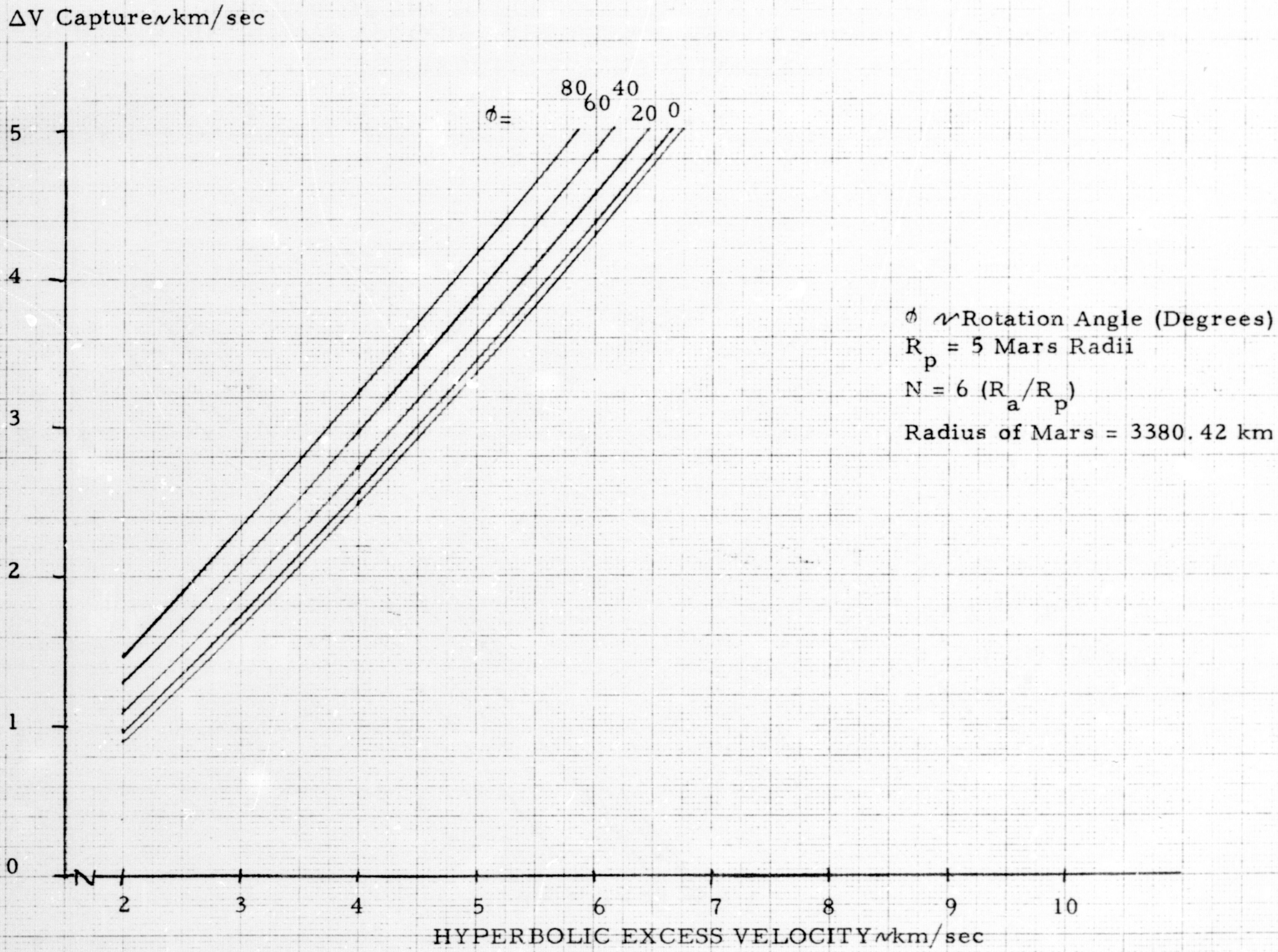

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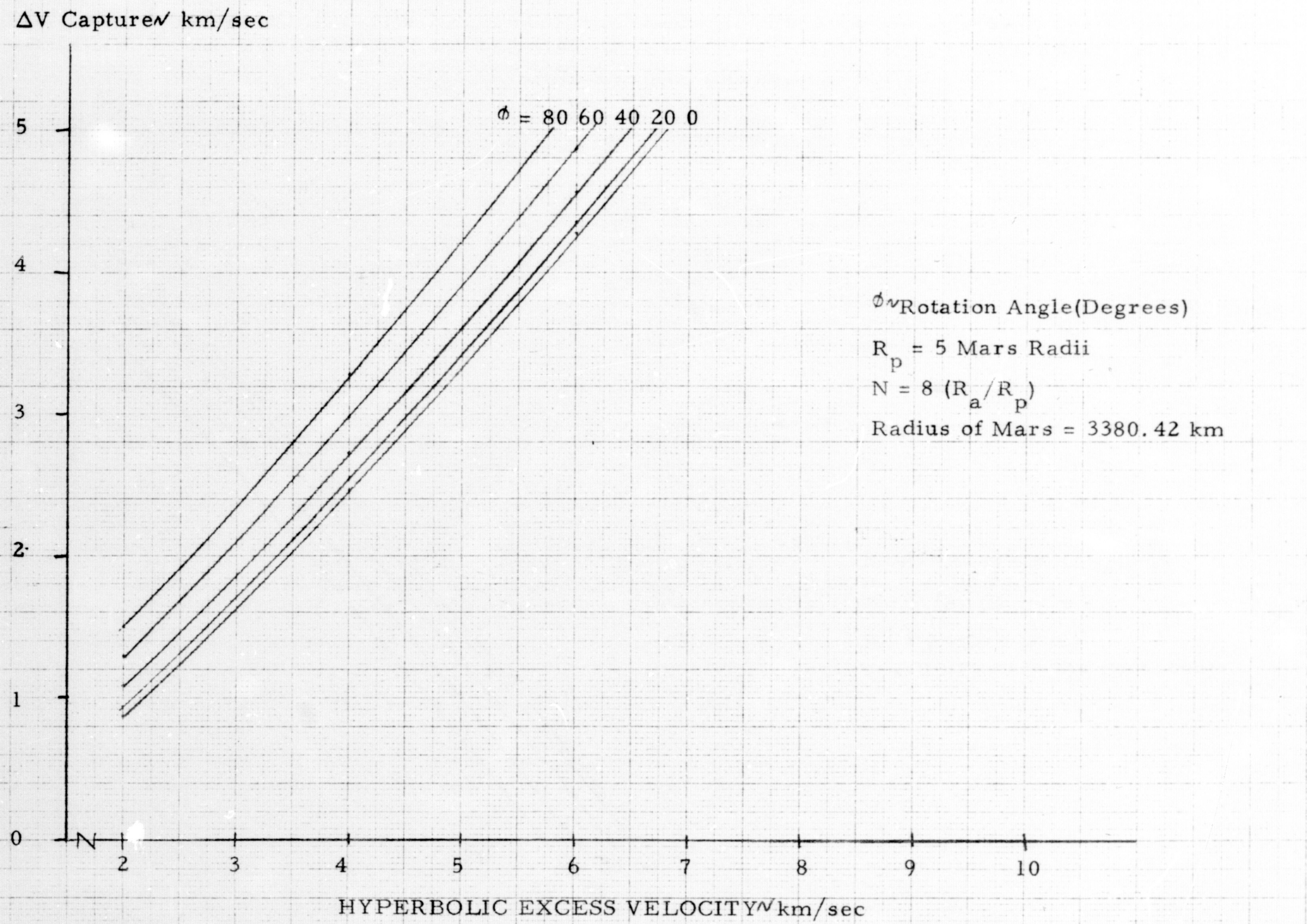

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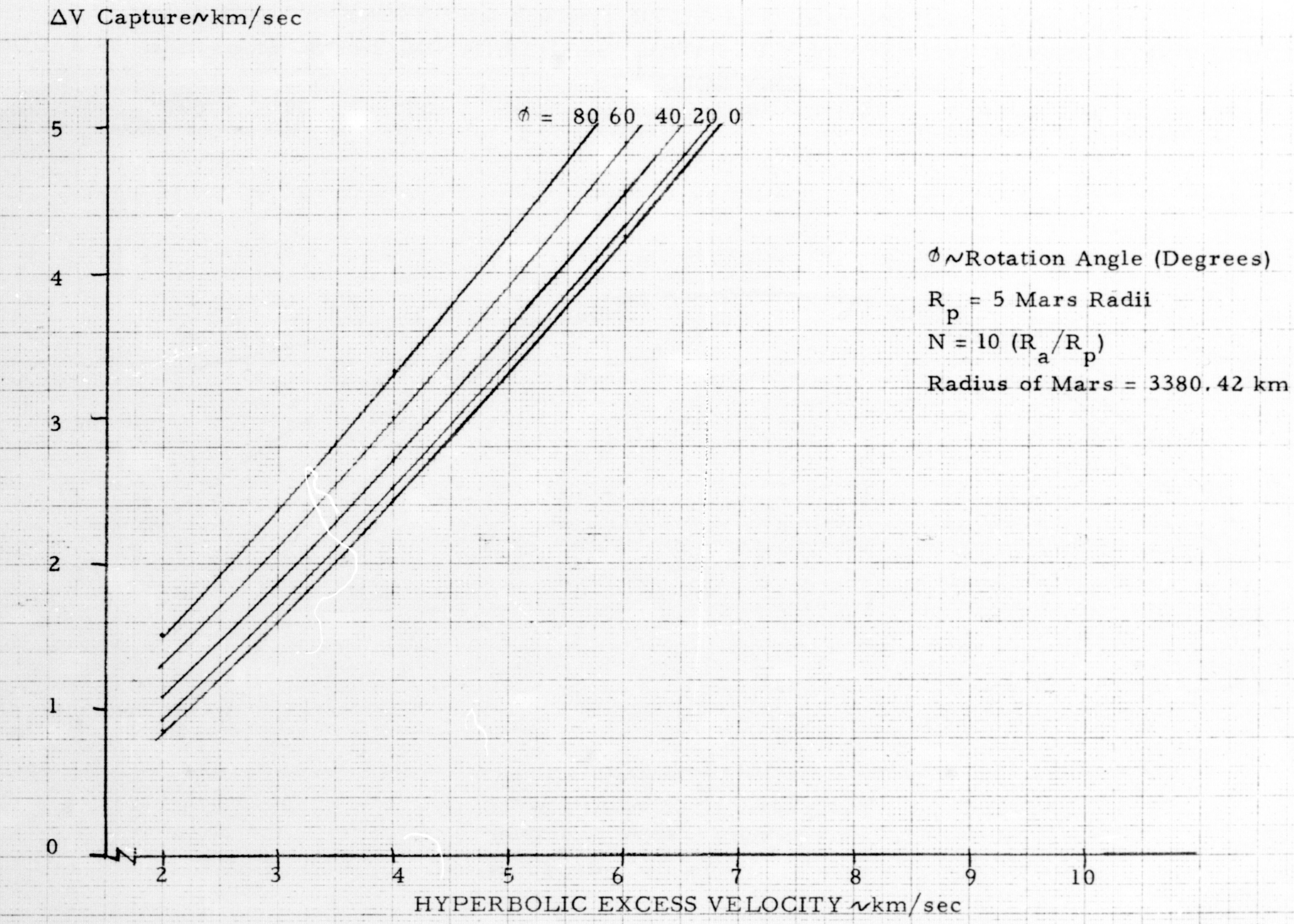

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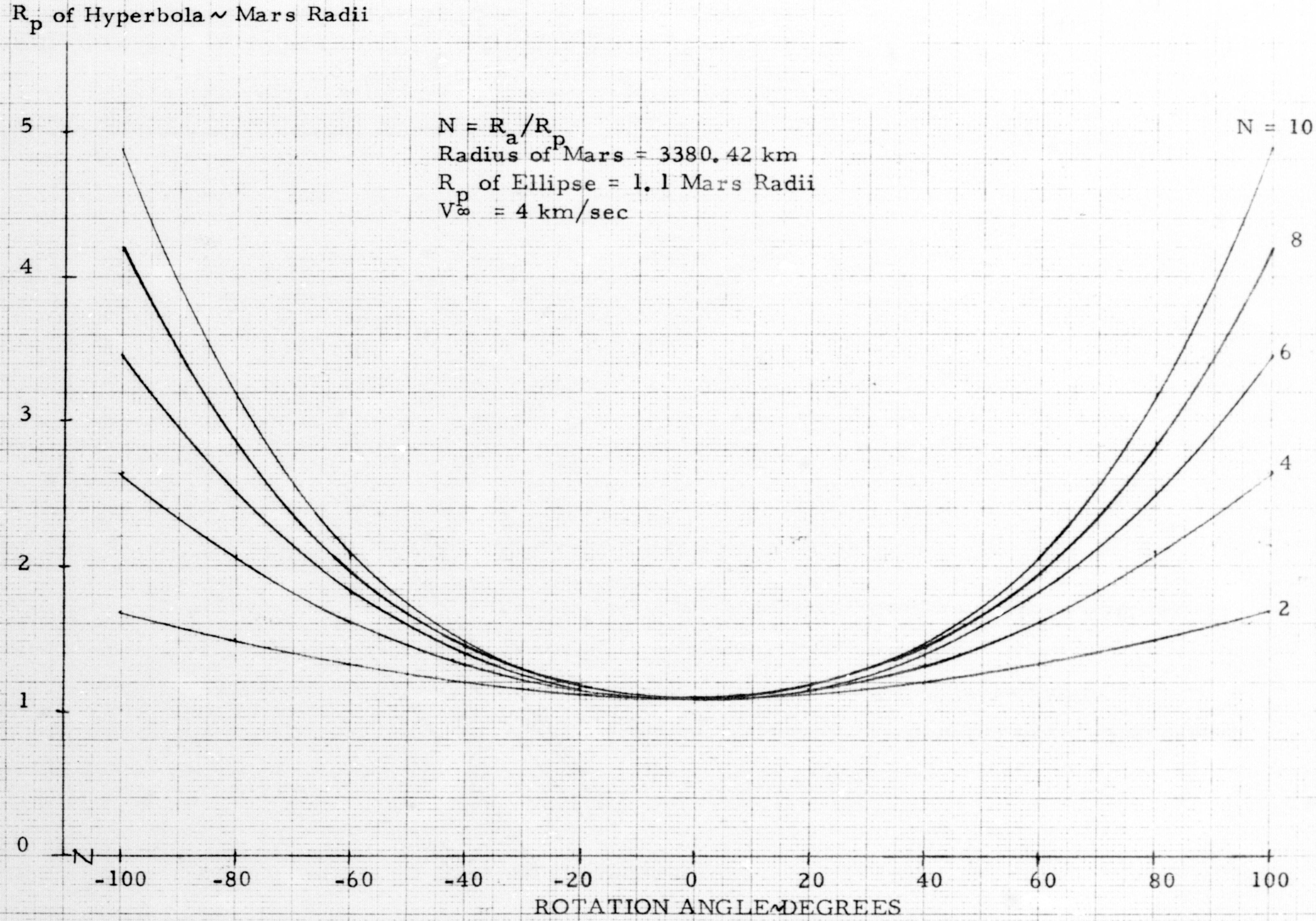


FIGURE 66 PERIAPSIS OF THE INCOMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO MARS ORBIT

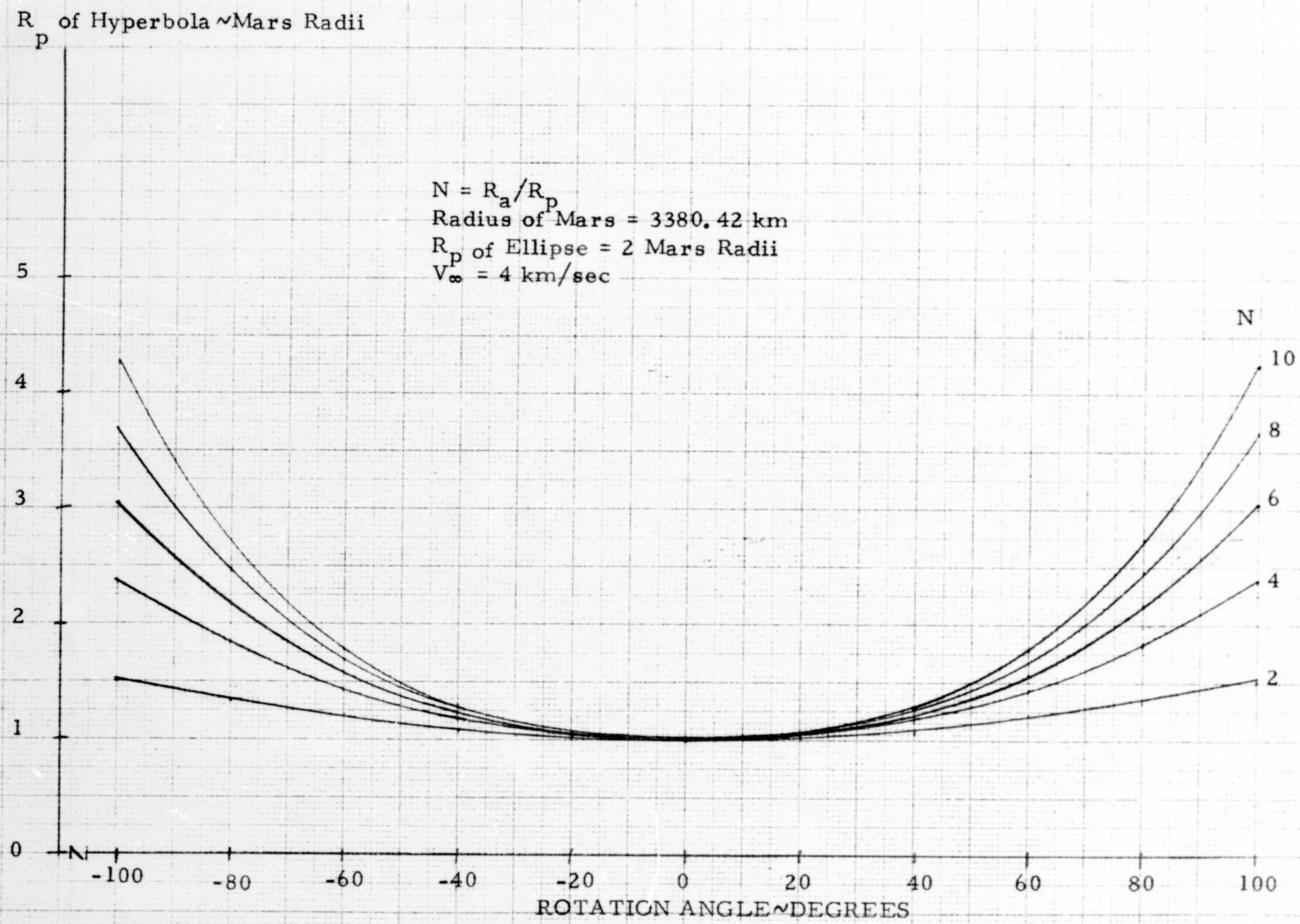


FIGURE 67 PERIAPSIS OF THE INCOMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO MARS ORBIT

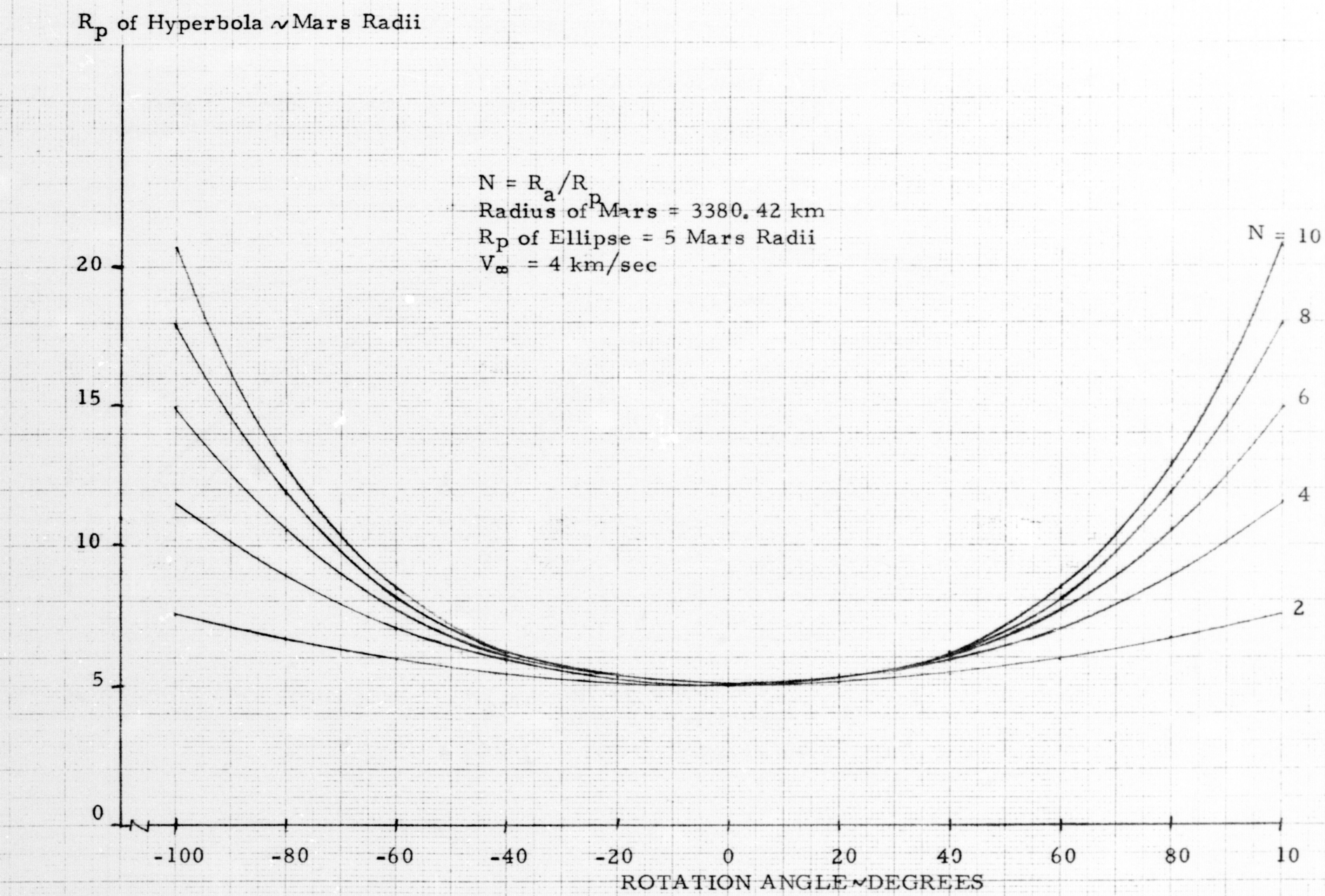


FIGURE 68 PERIAPSIS OF THE INCOMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO MARS ORBIT

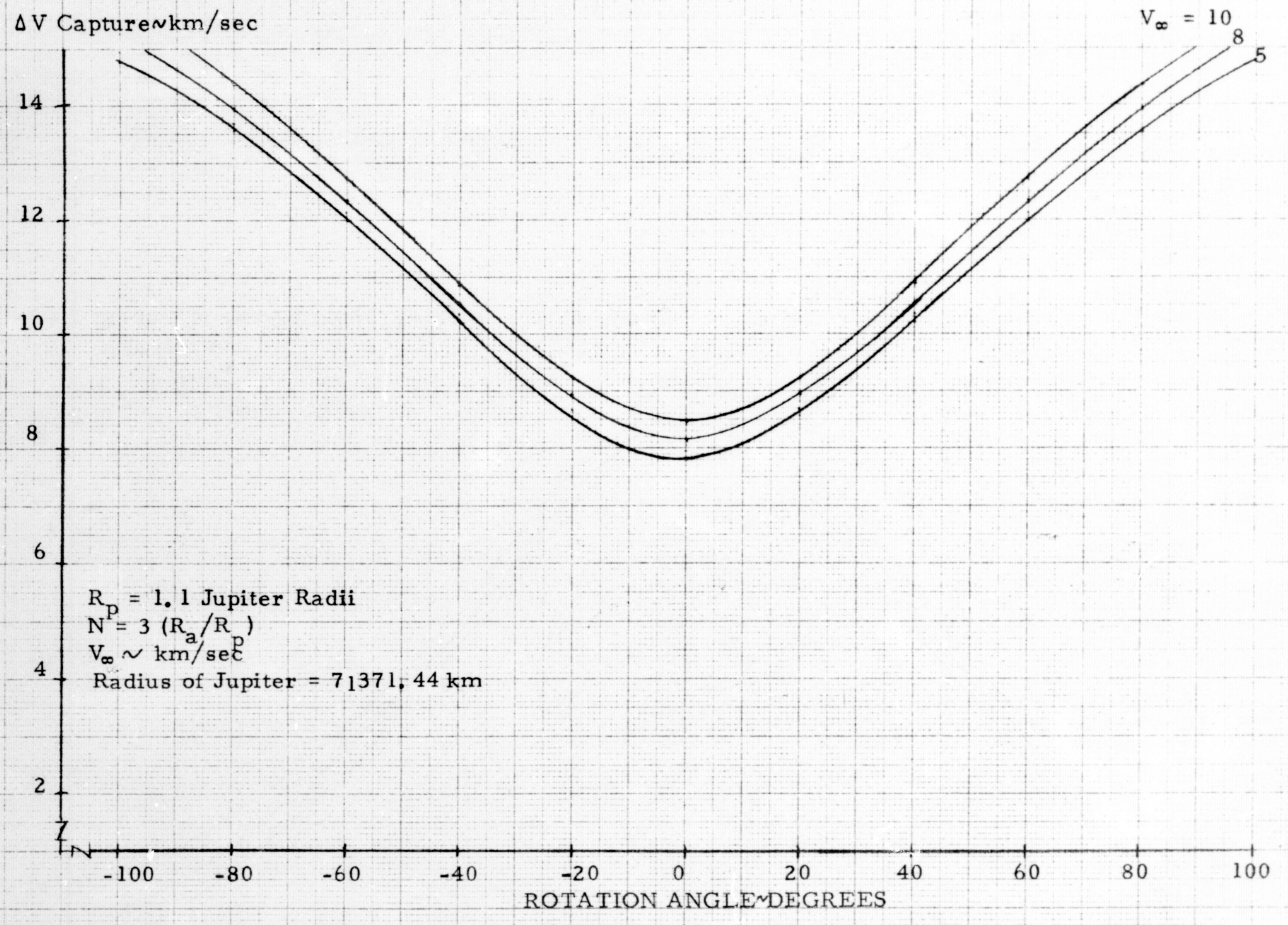


FIGURE 69 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF ROTATION ANGLE(ROT) FOR TANGENTIAL TRANSFER

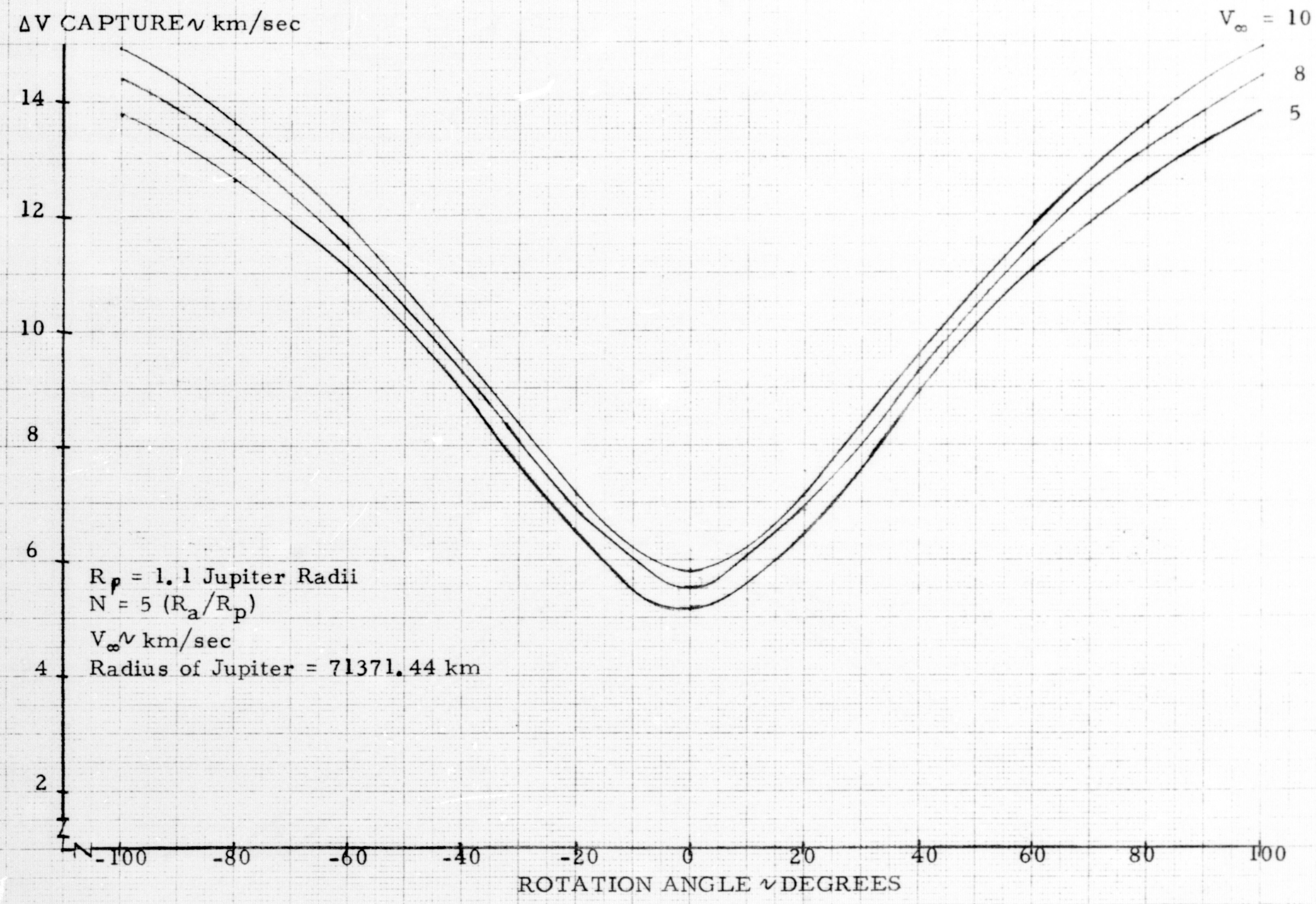
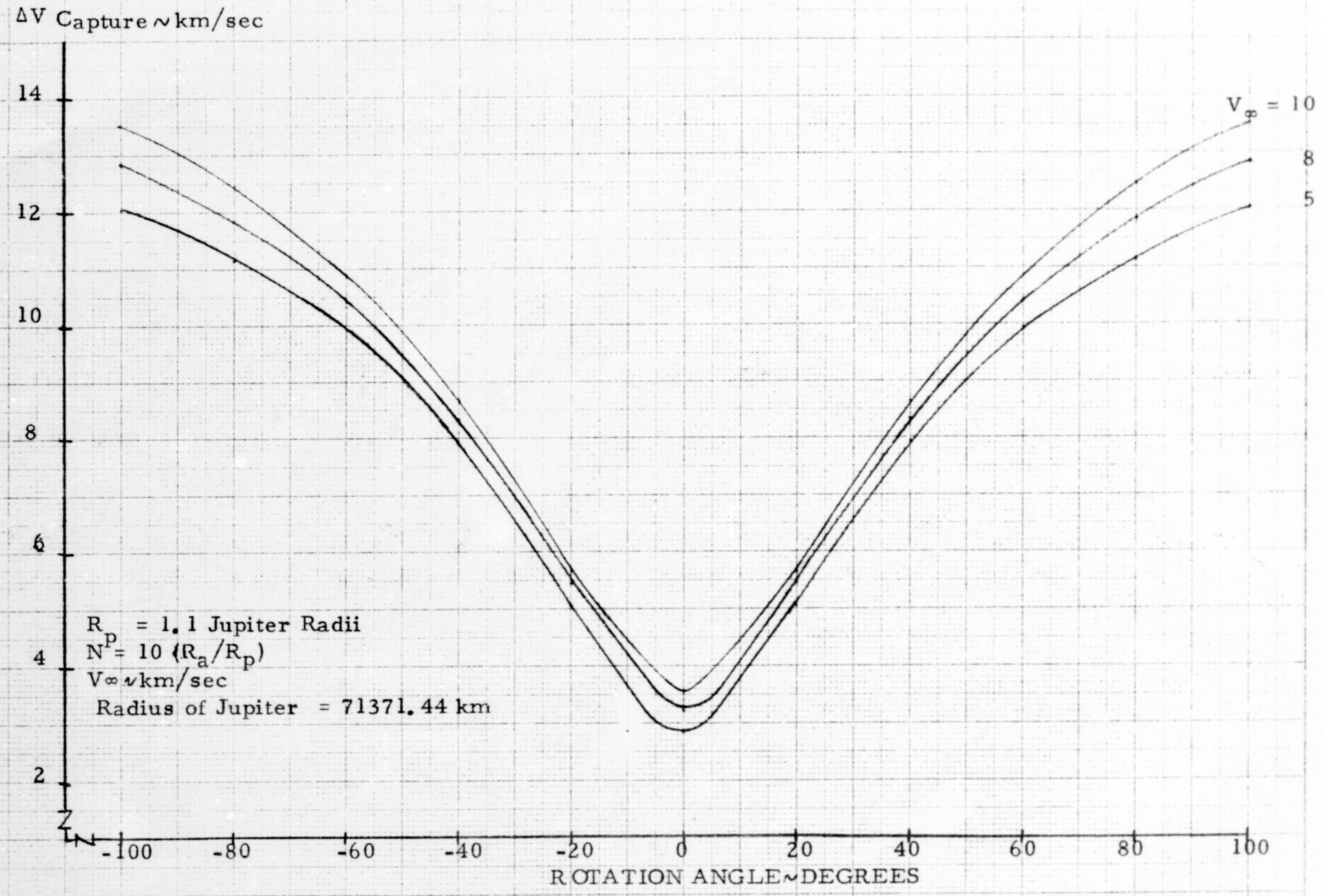


FIGURE 70 IMPULSVIE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBITS ABOUT JUPITER AS AS FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER



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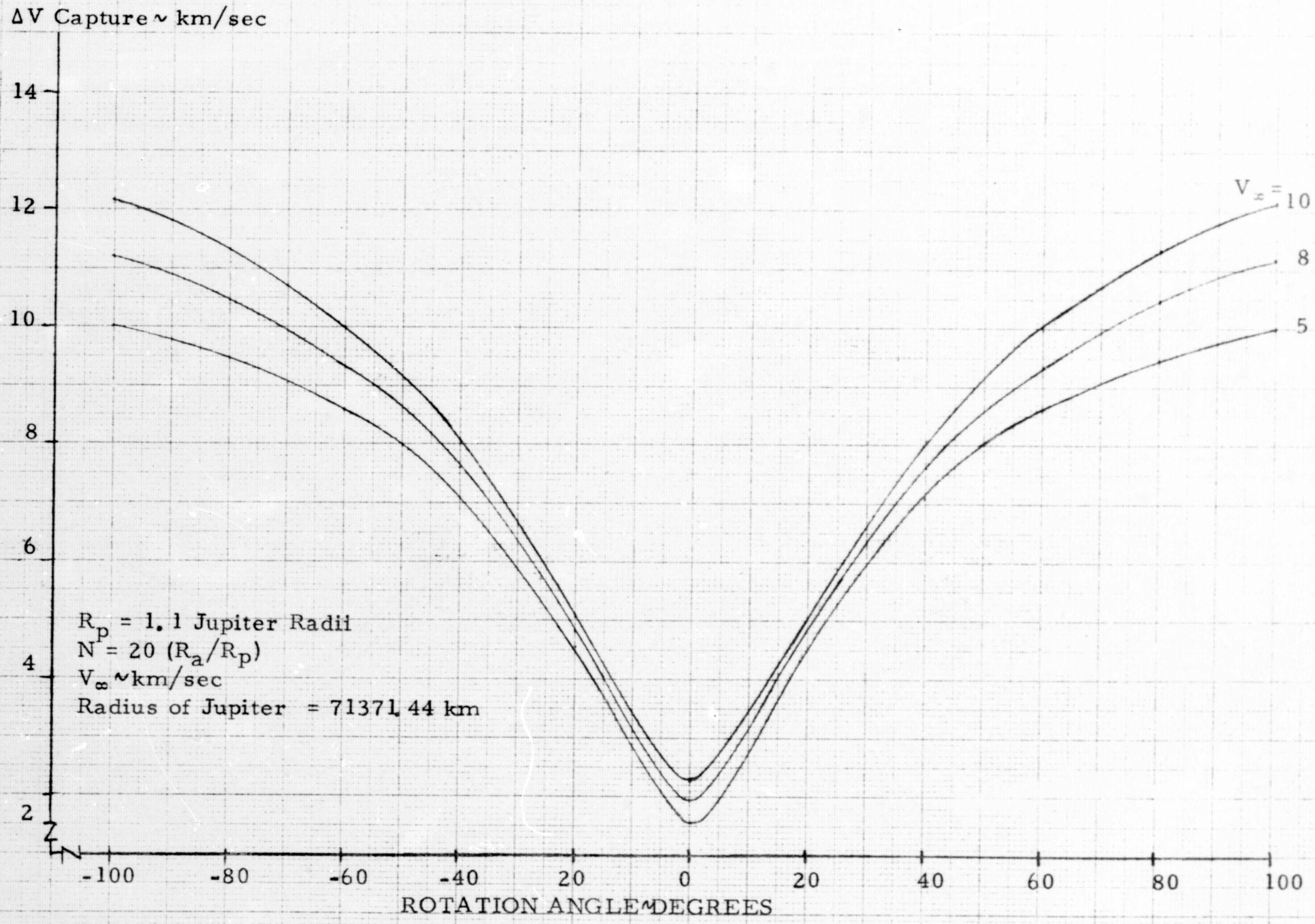

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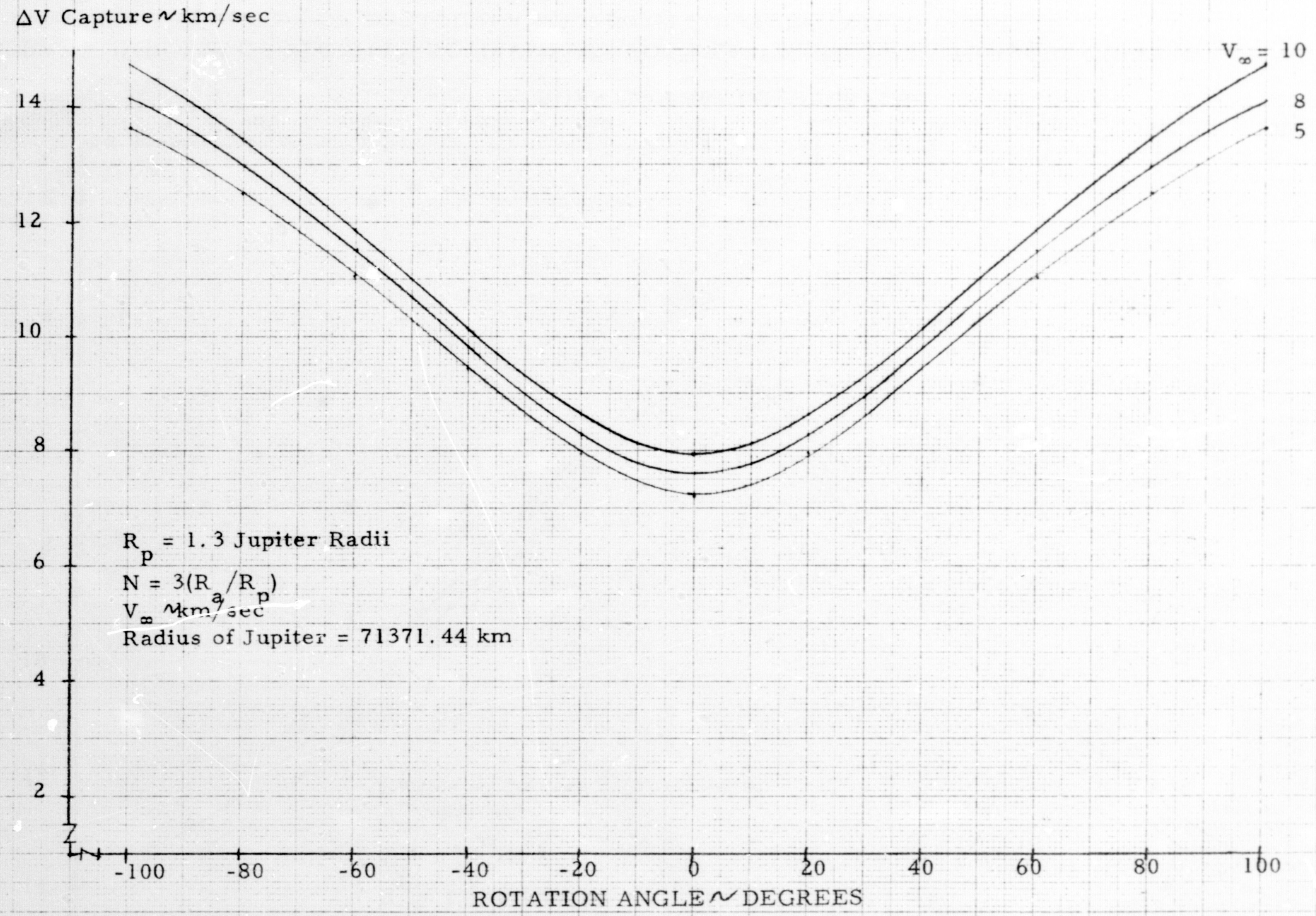


FIGURE 73 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBITS ABOUT JUPITER AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

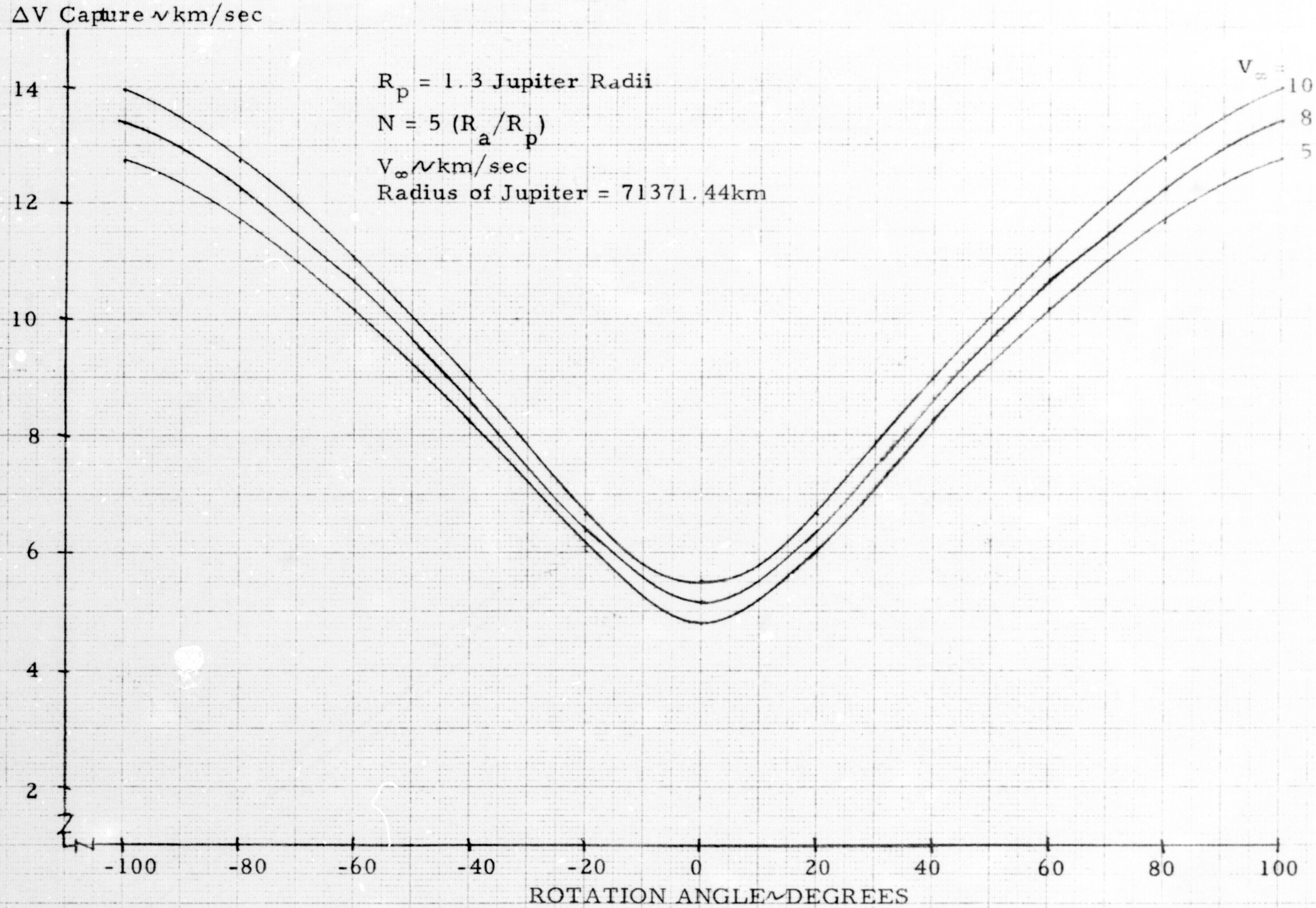


FIGURE 74 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBITS ABOUT JUPITER AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

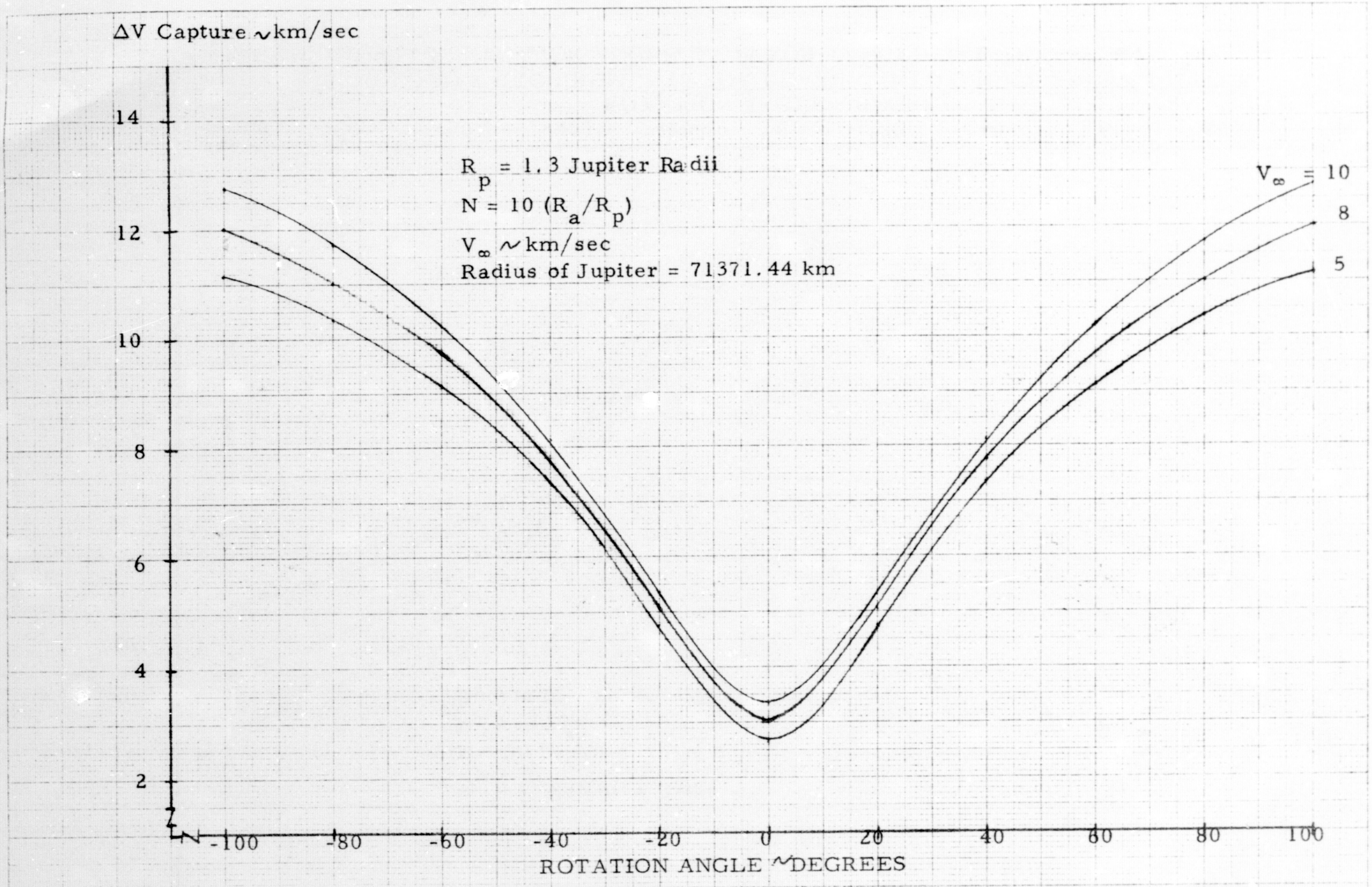
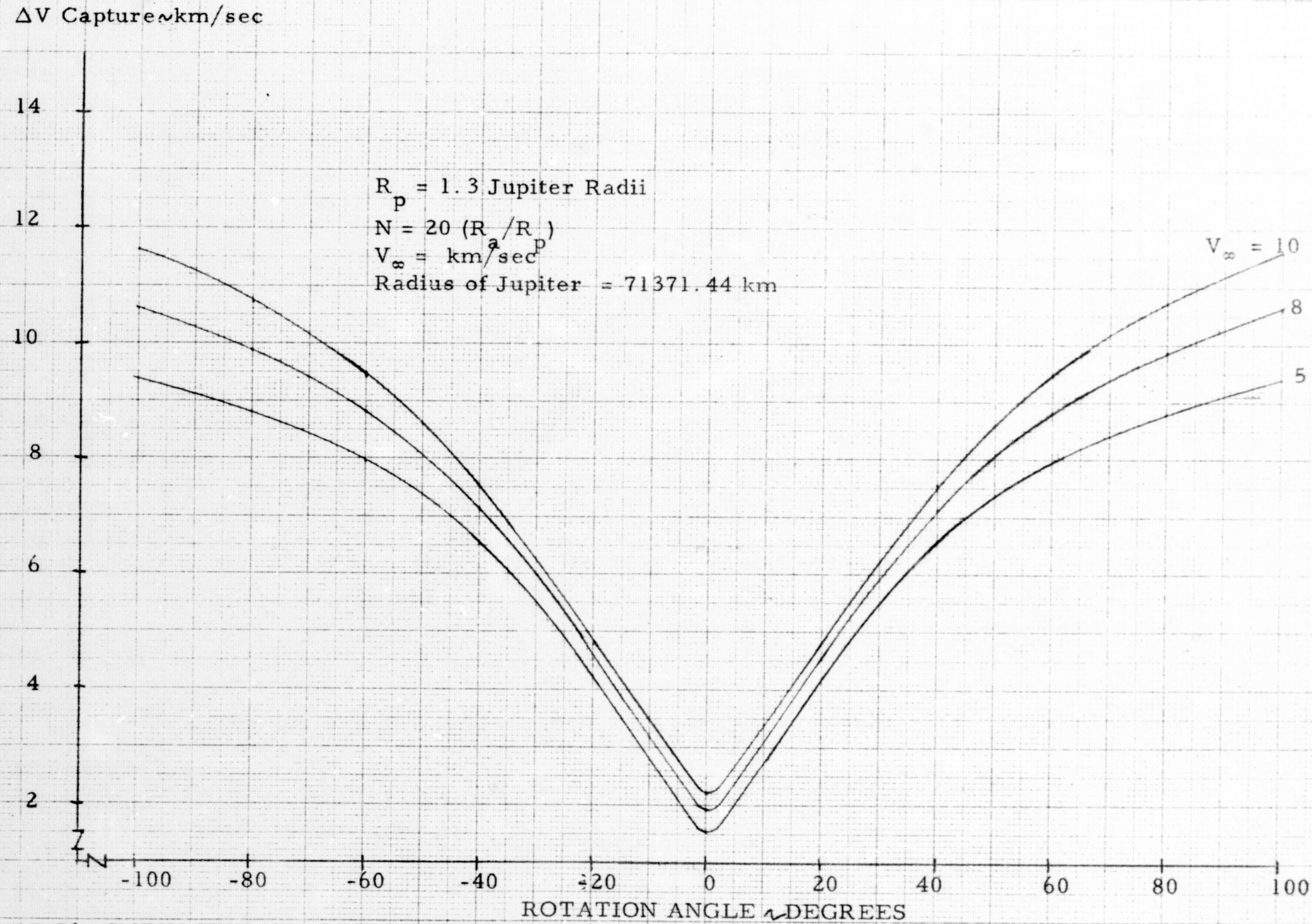


FIGURE 75 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBITS ABOUT JUPITER AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER



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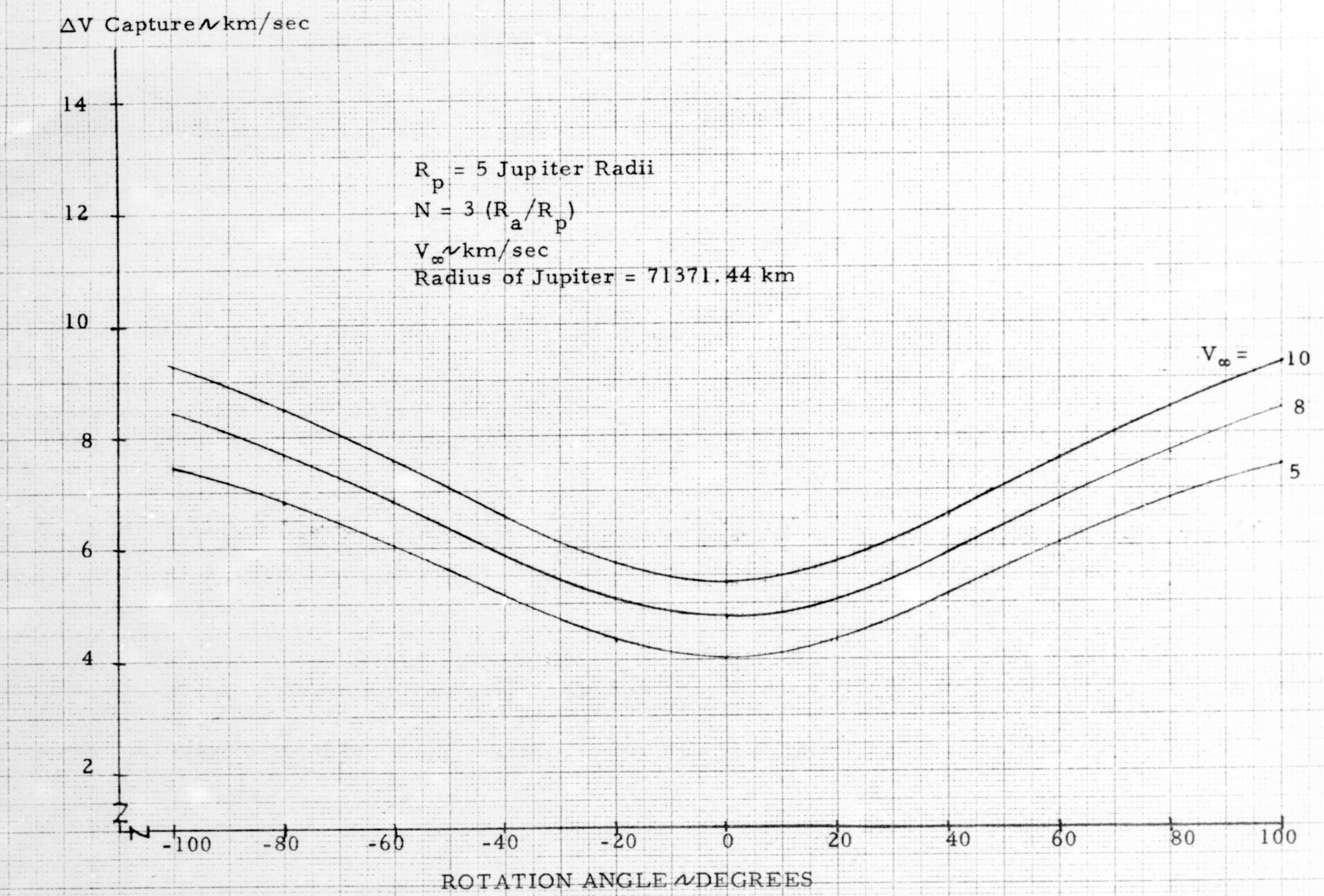

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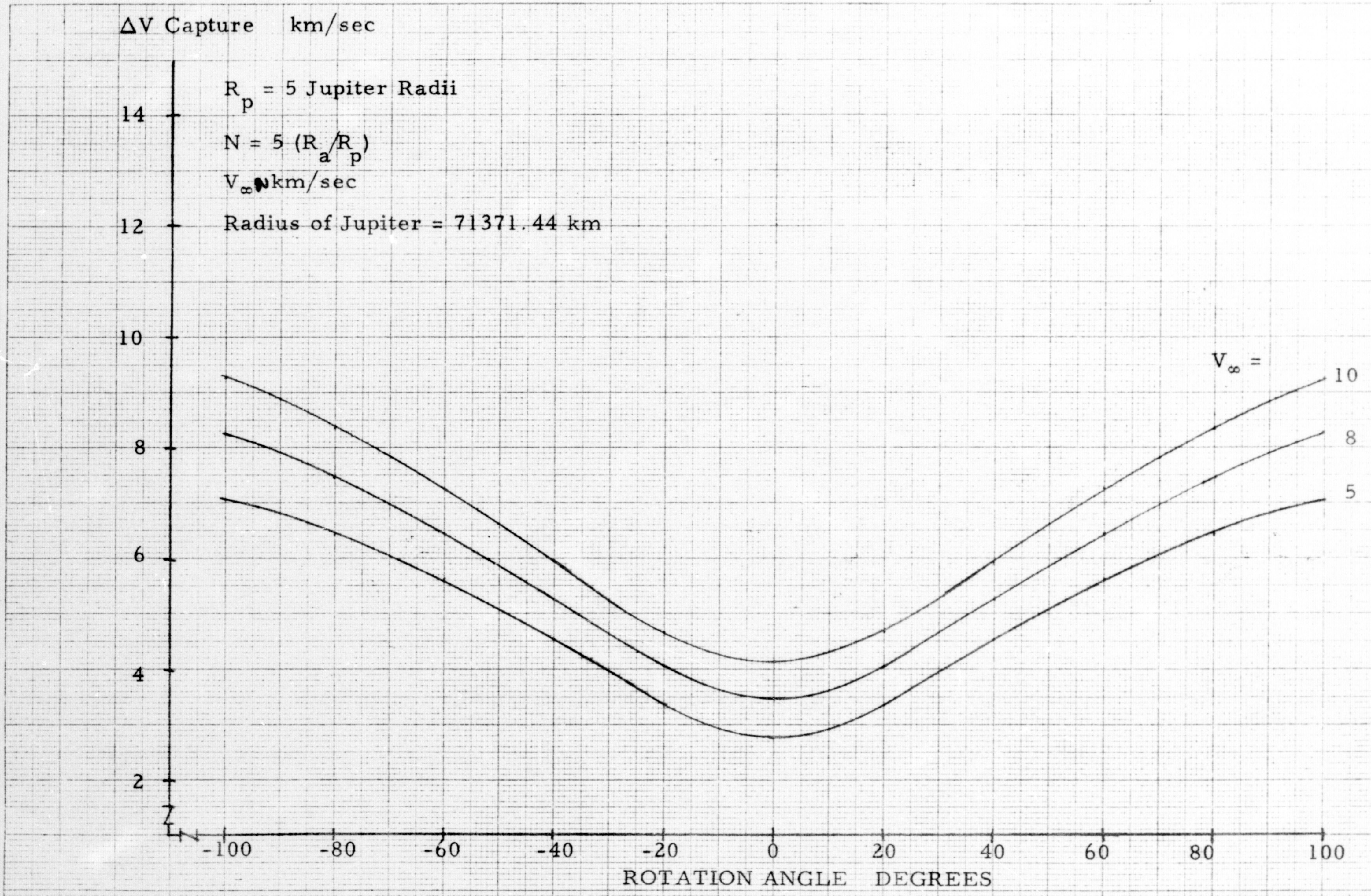

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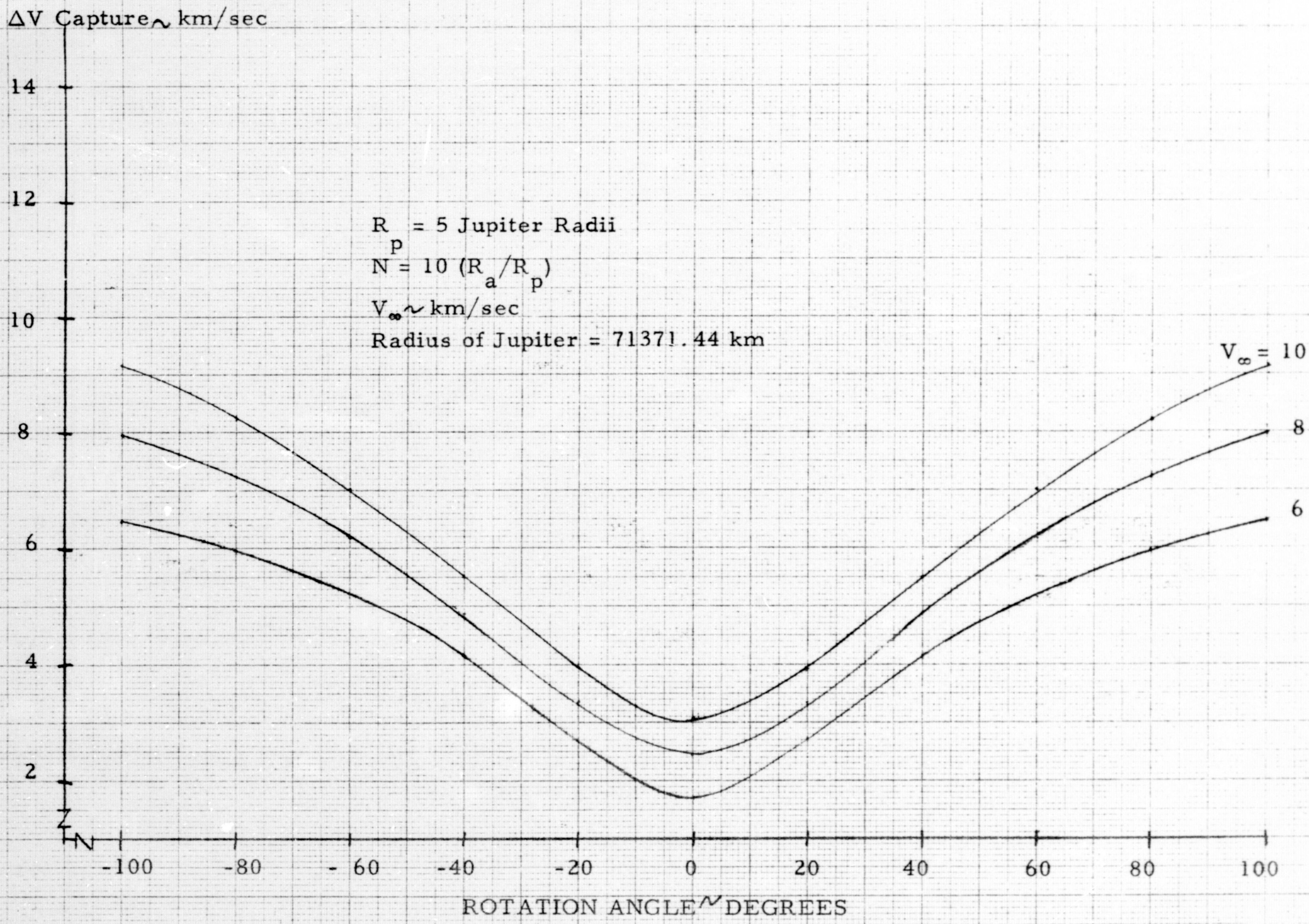

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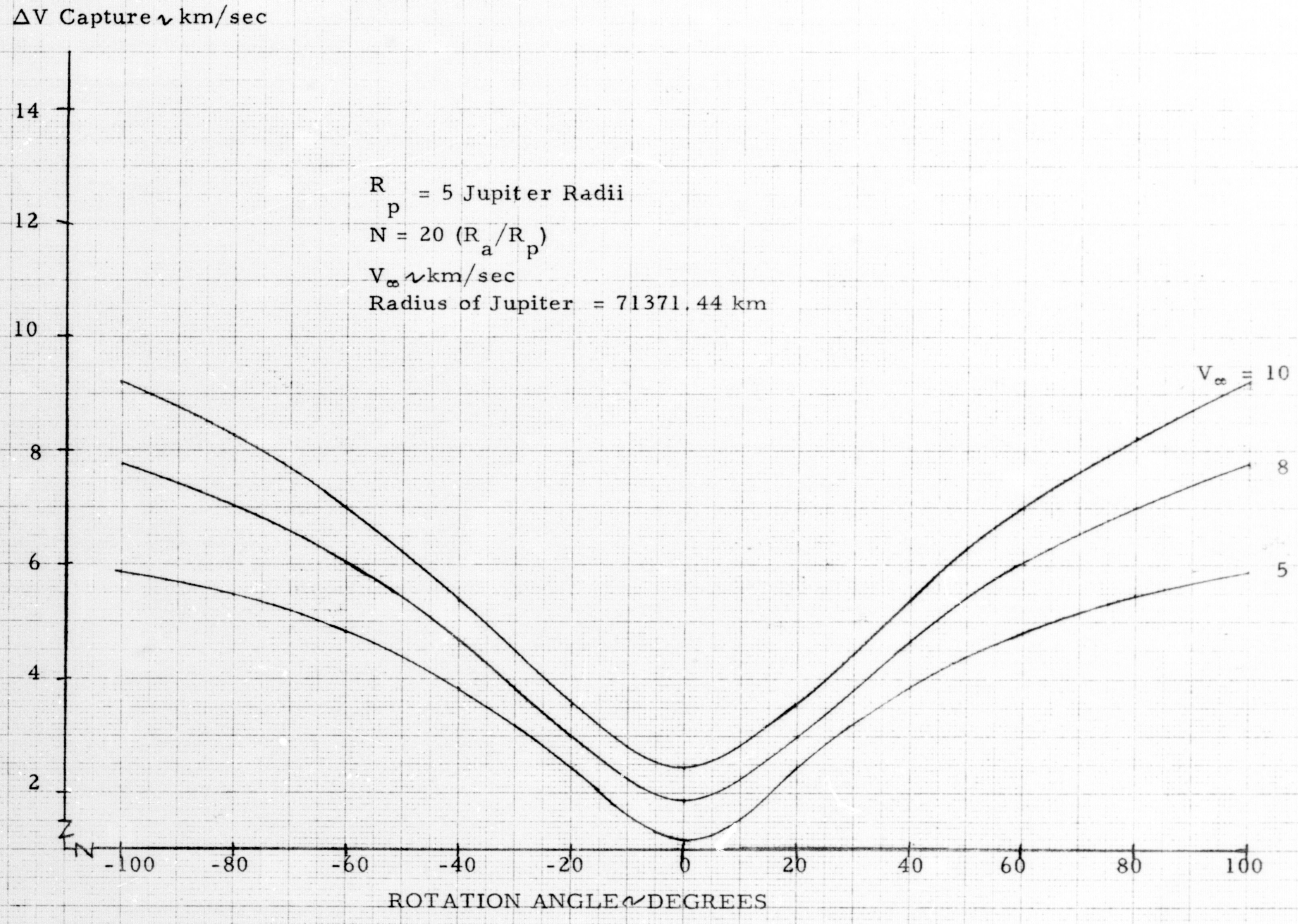

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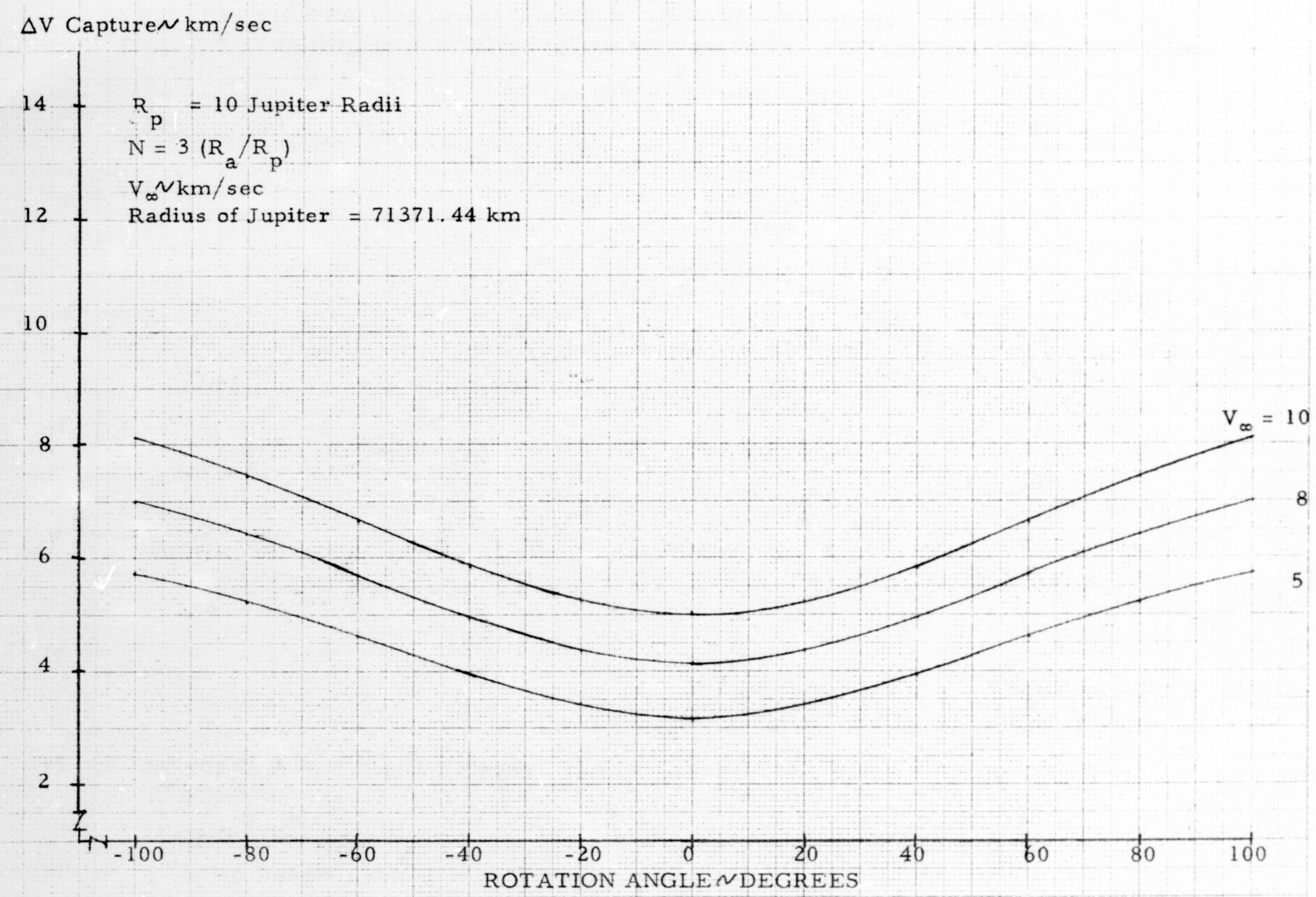

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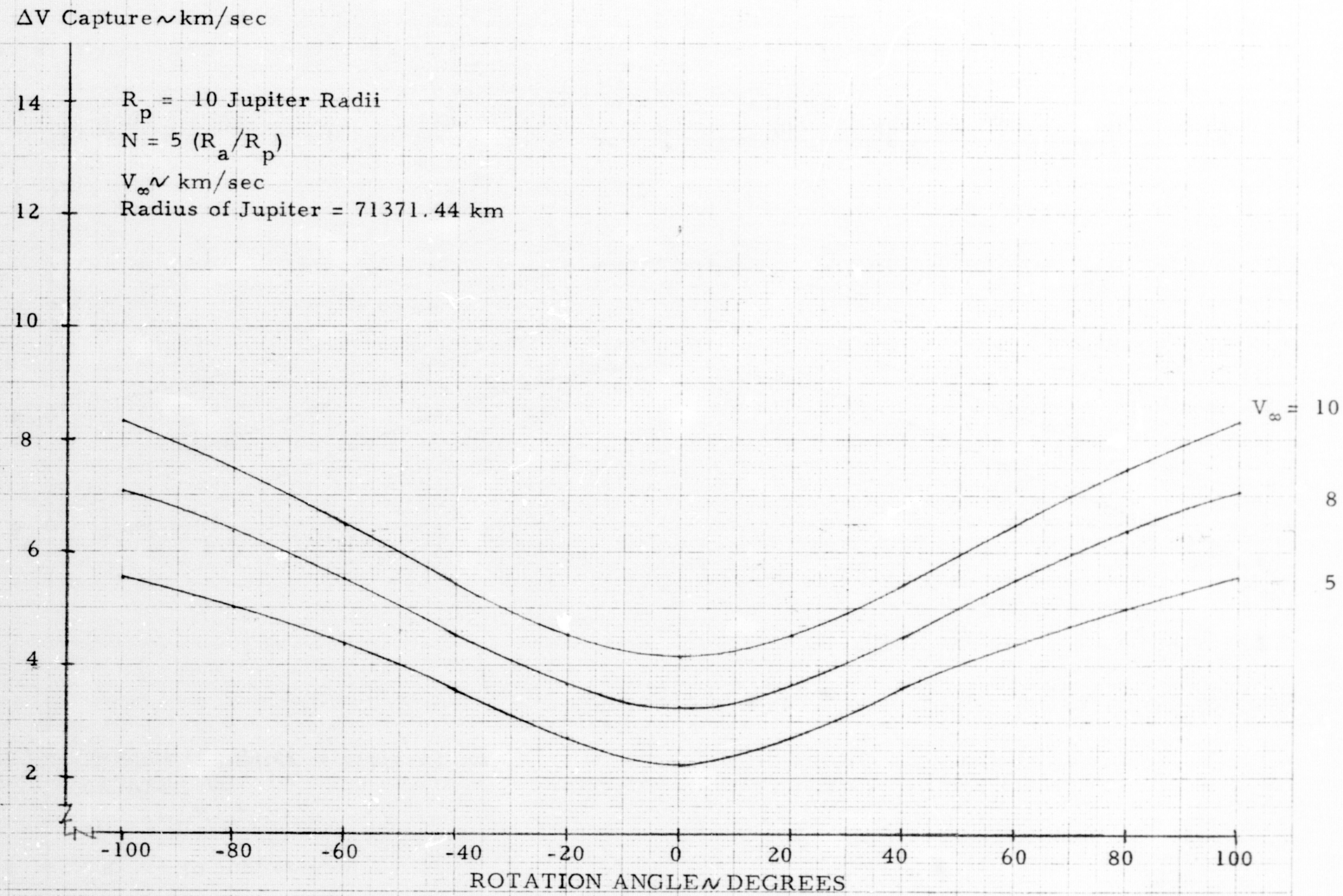

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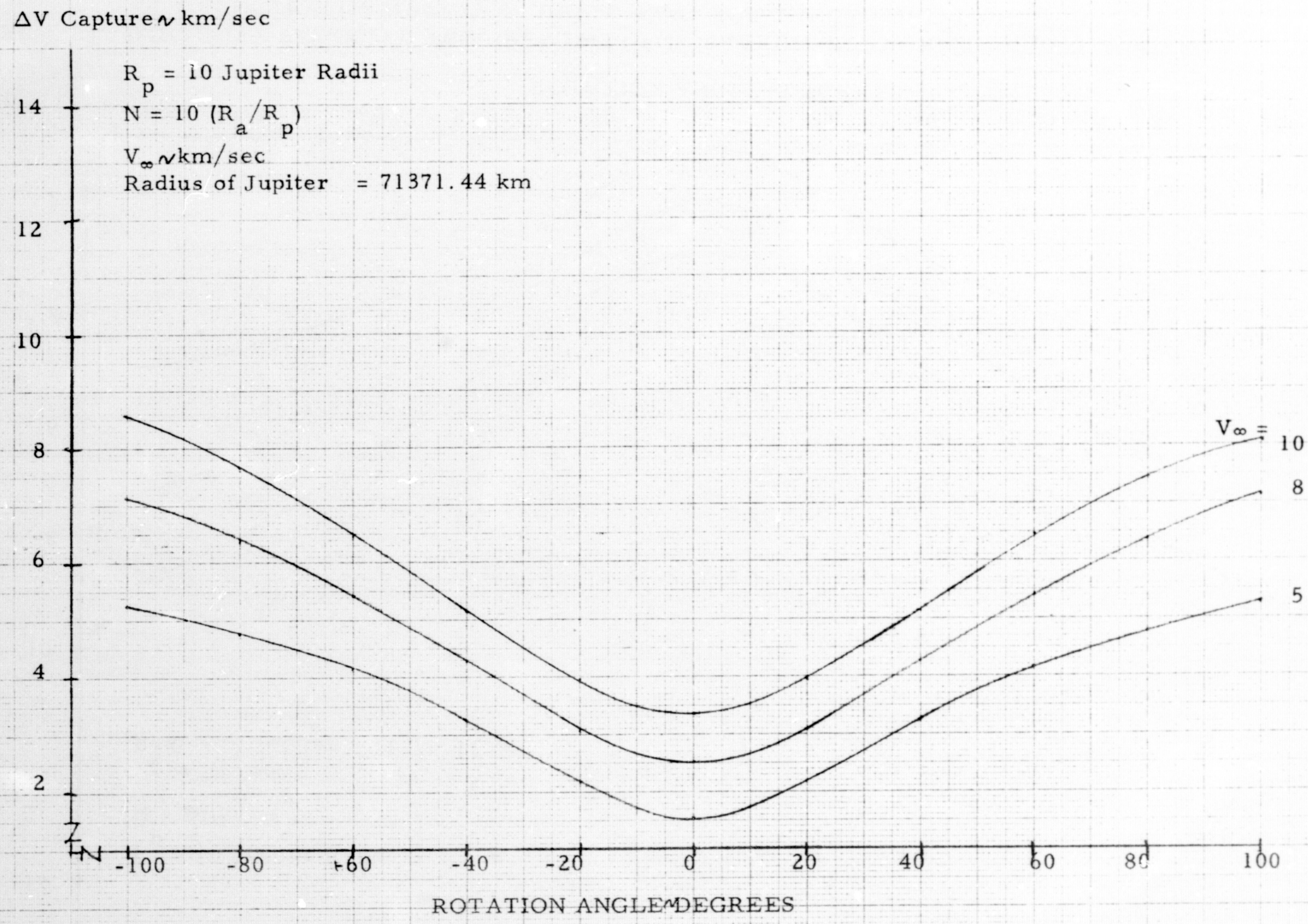

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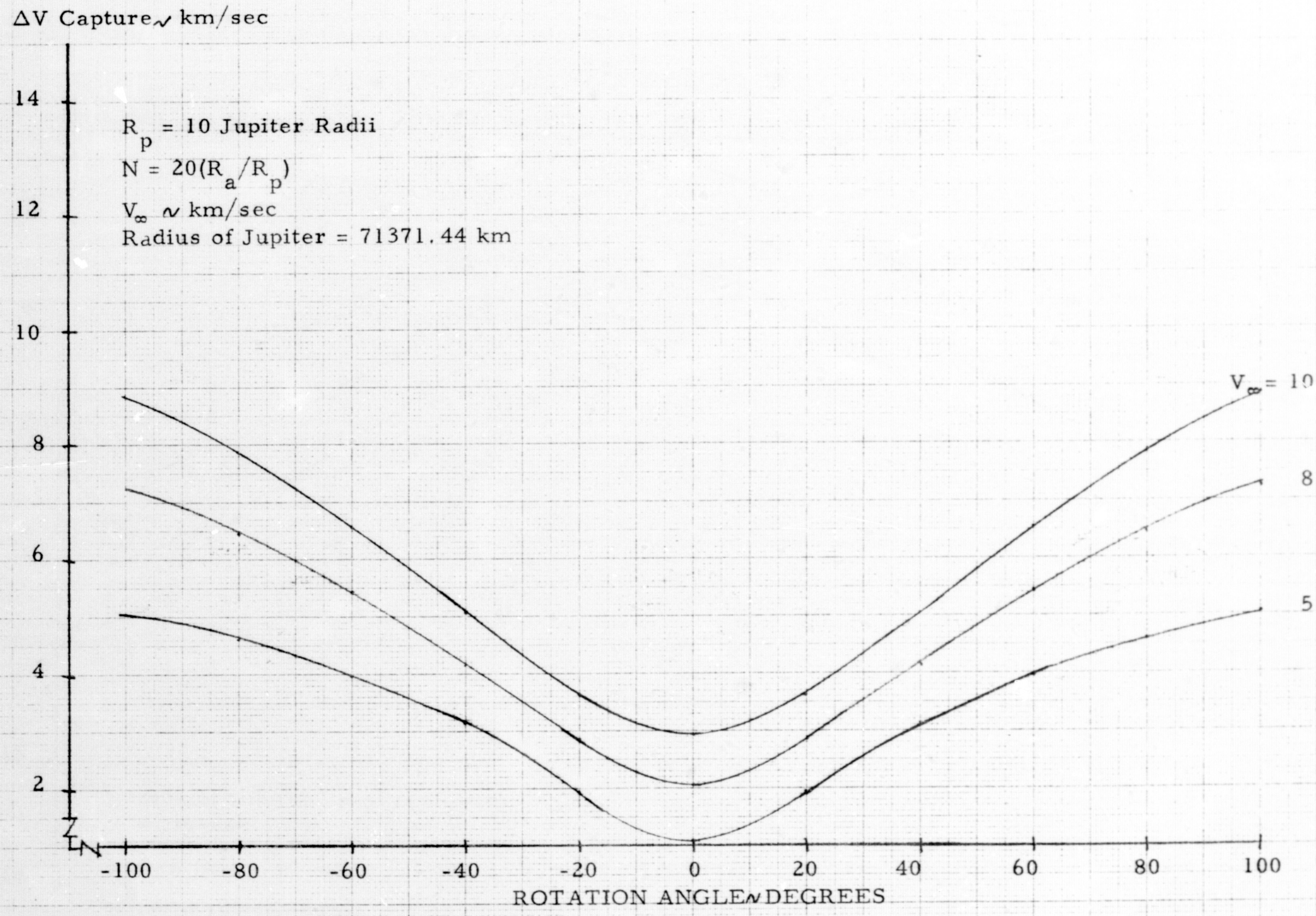

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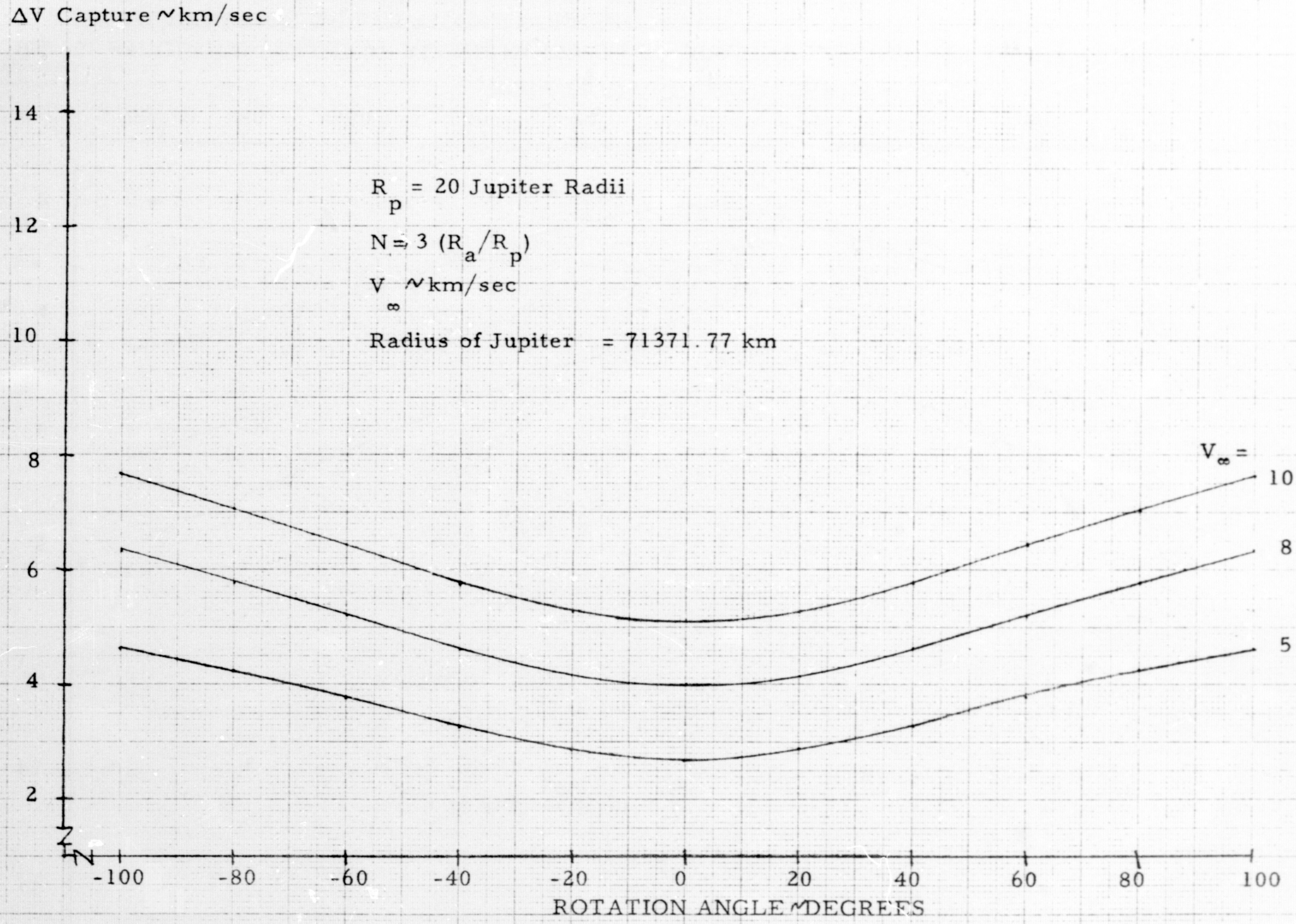

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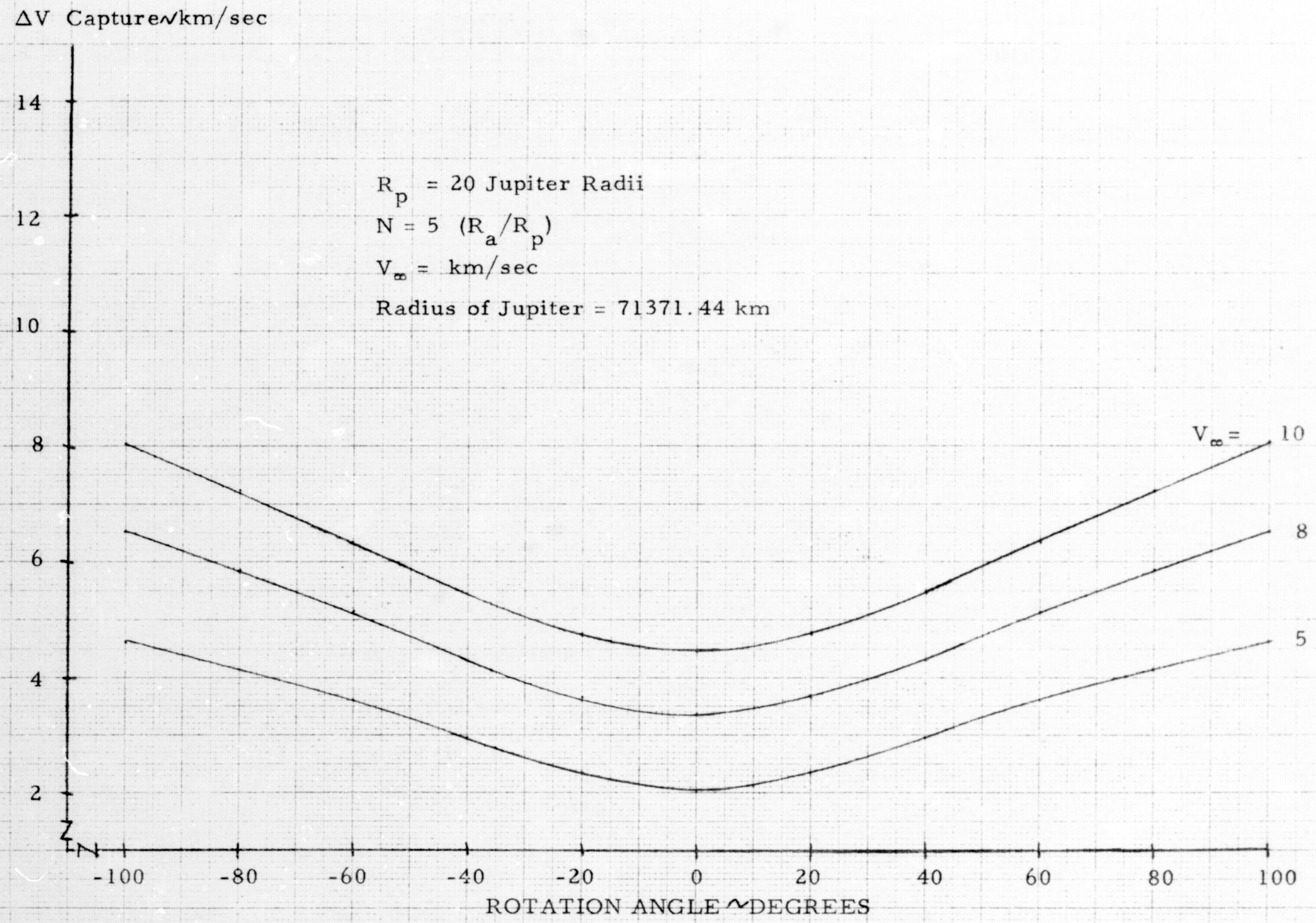

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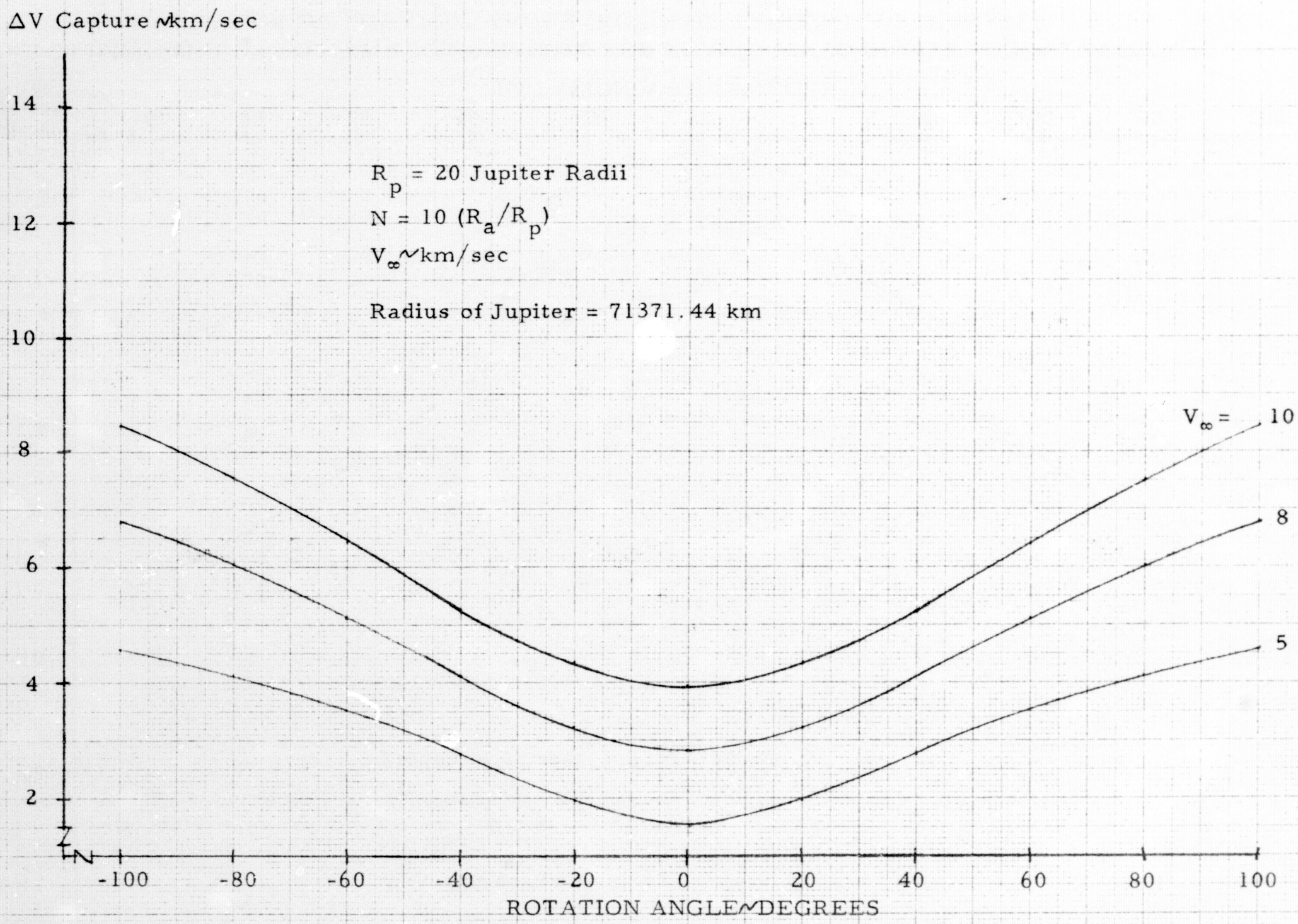

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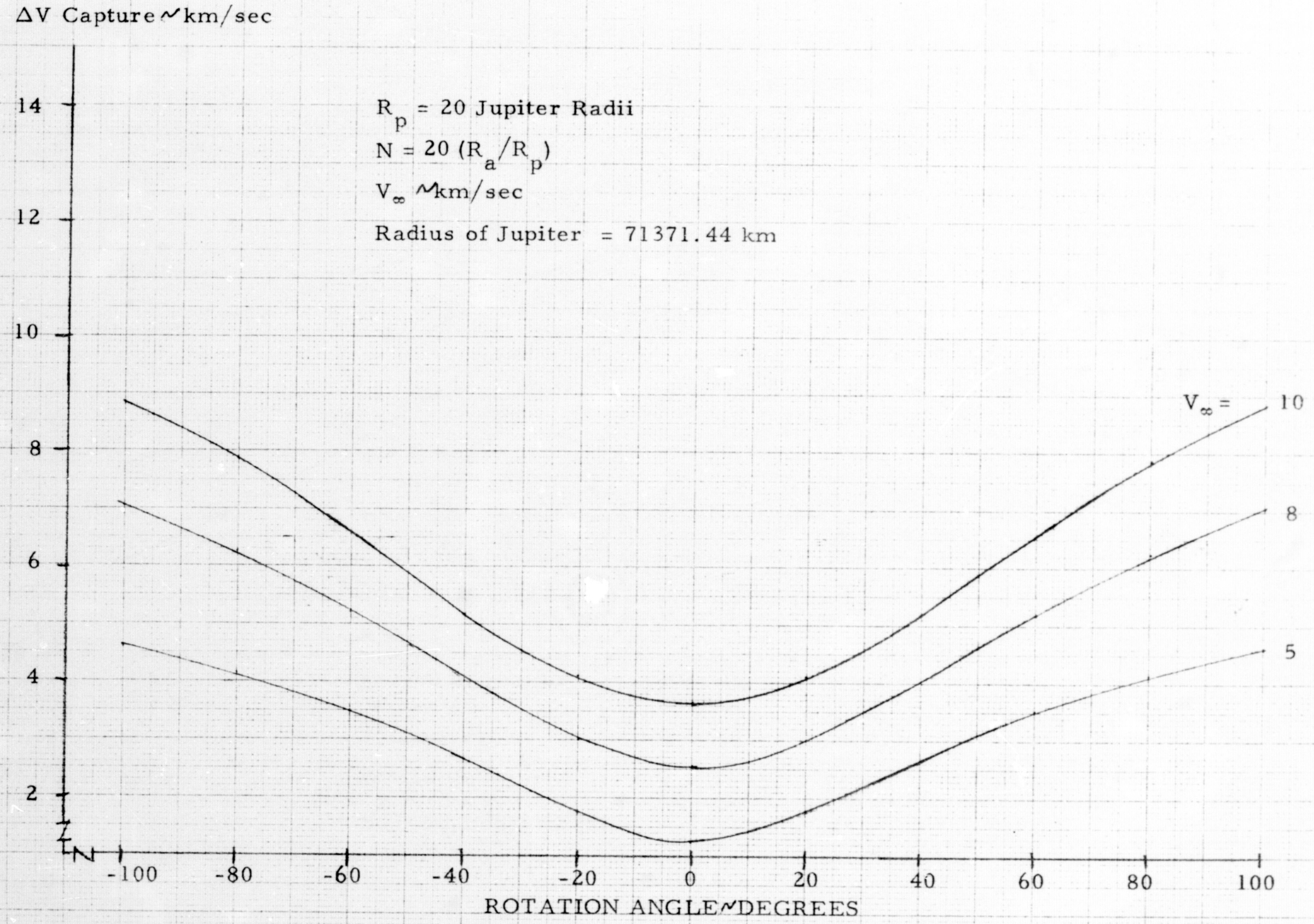


FIGURE 88 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBITS ABOUT JUPITER AS A FUNCTION OF ROTATION ANGLE (ROT) FOR TANGENTIAL TRANSFER

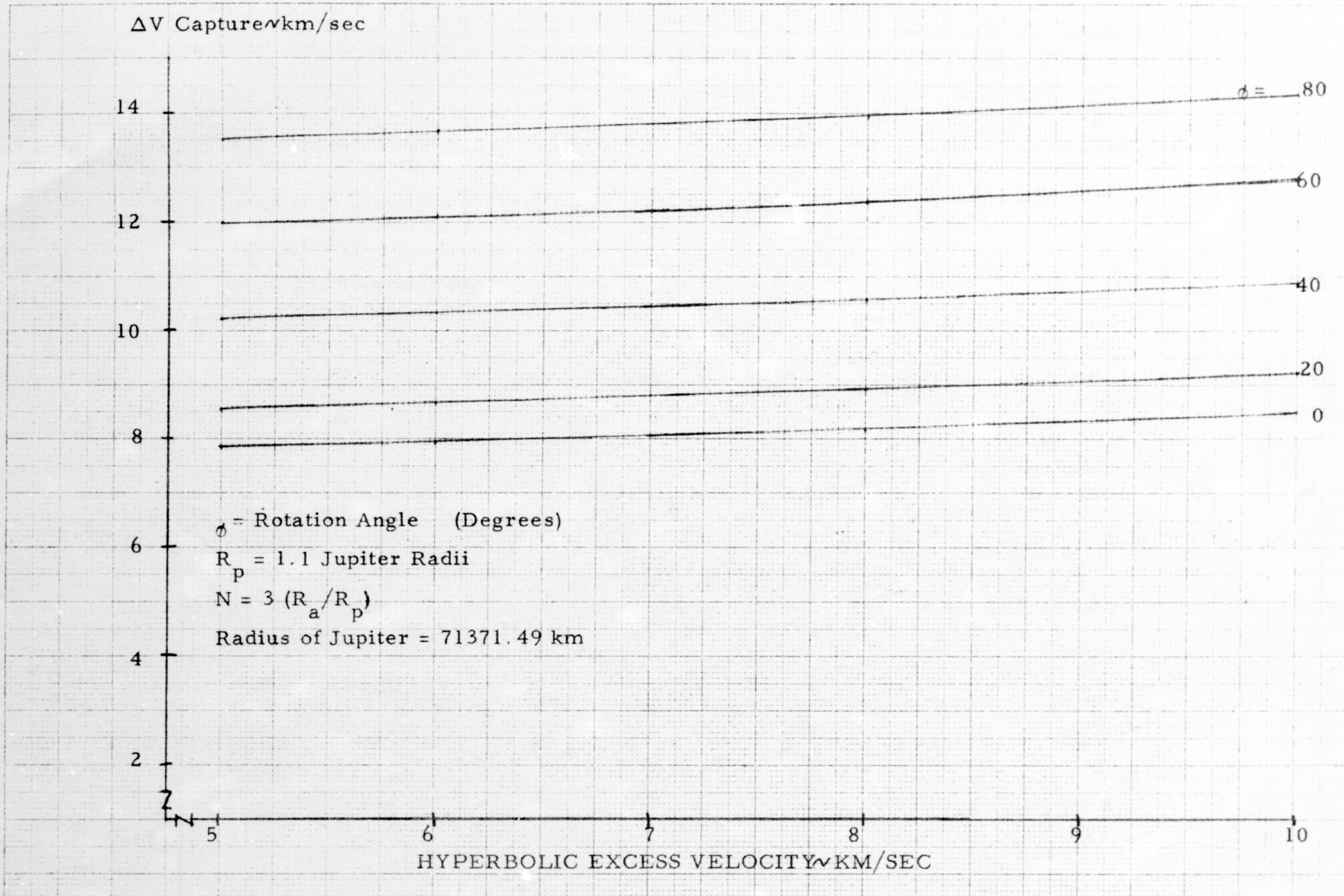


FIGURE 89 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) for TANGENTIAL TRANSFER

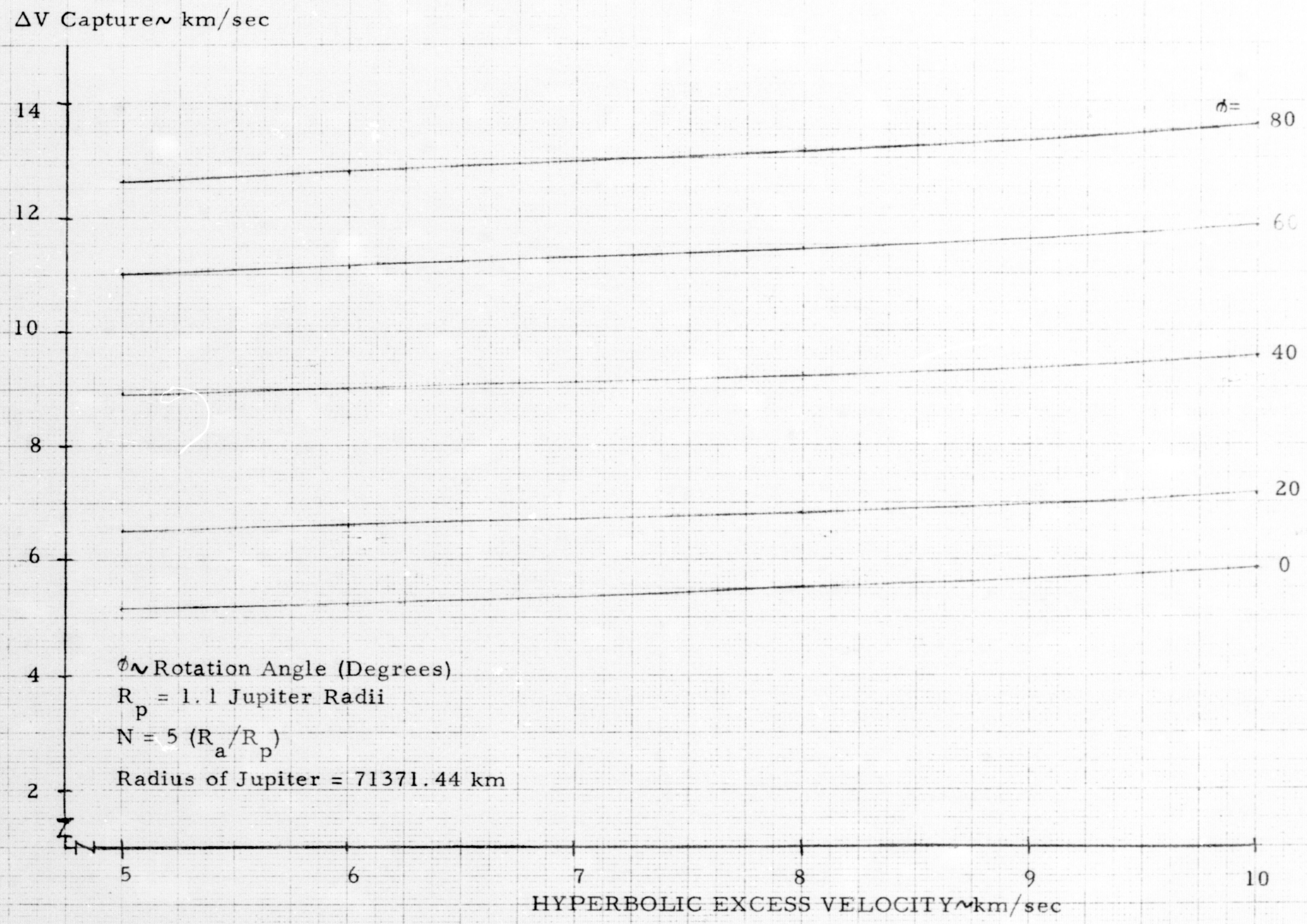


FIGURE 90 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

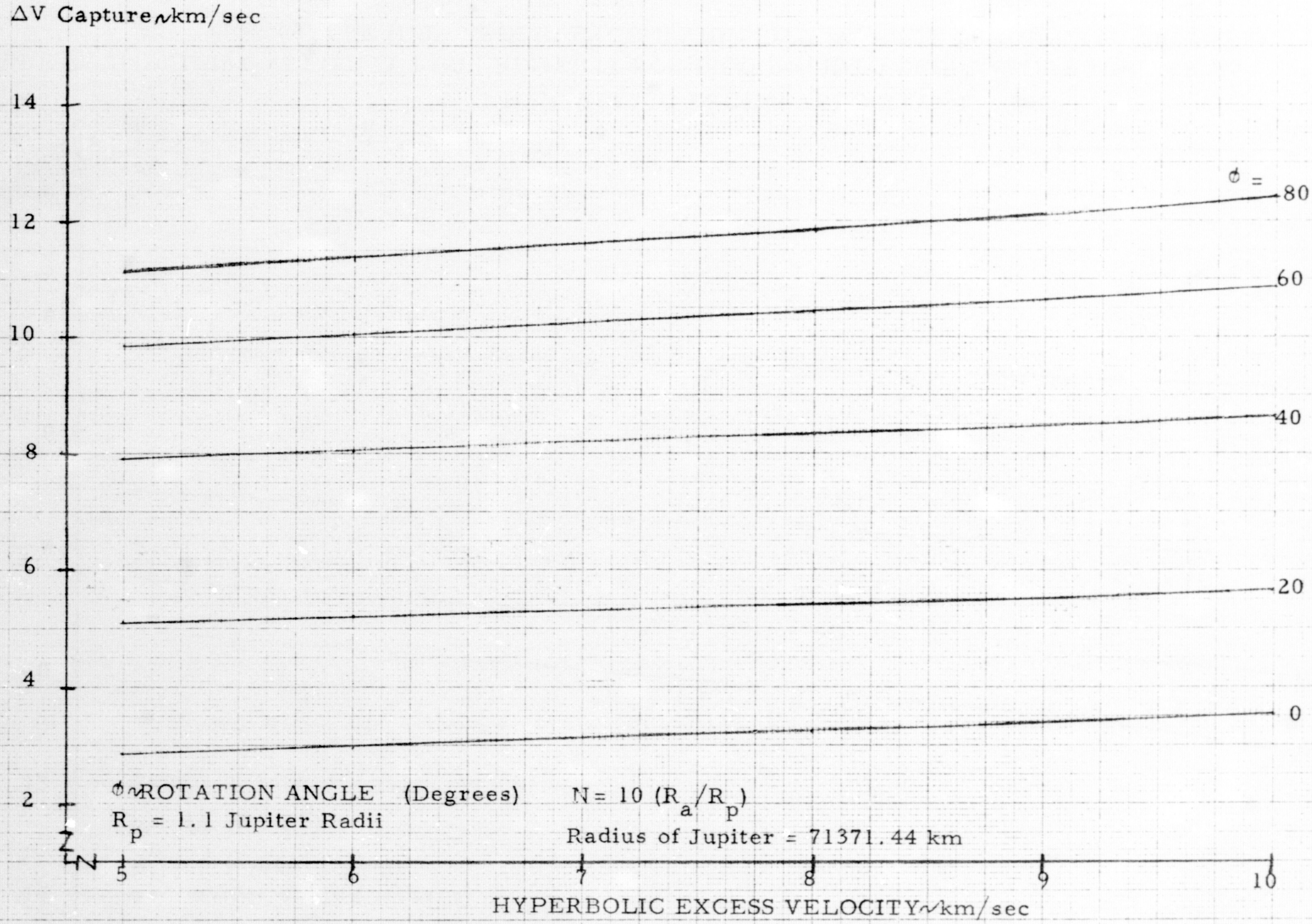
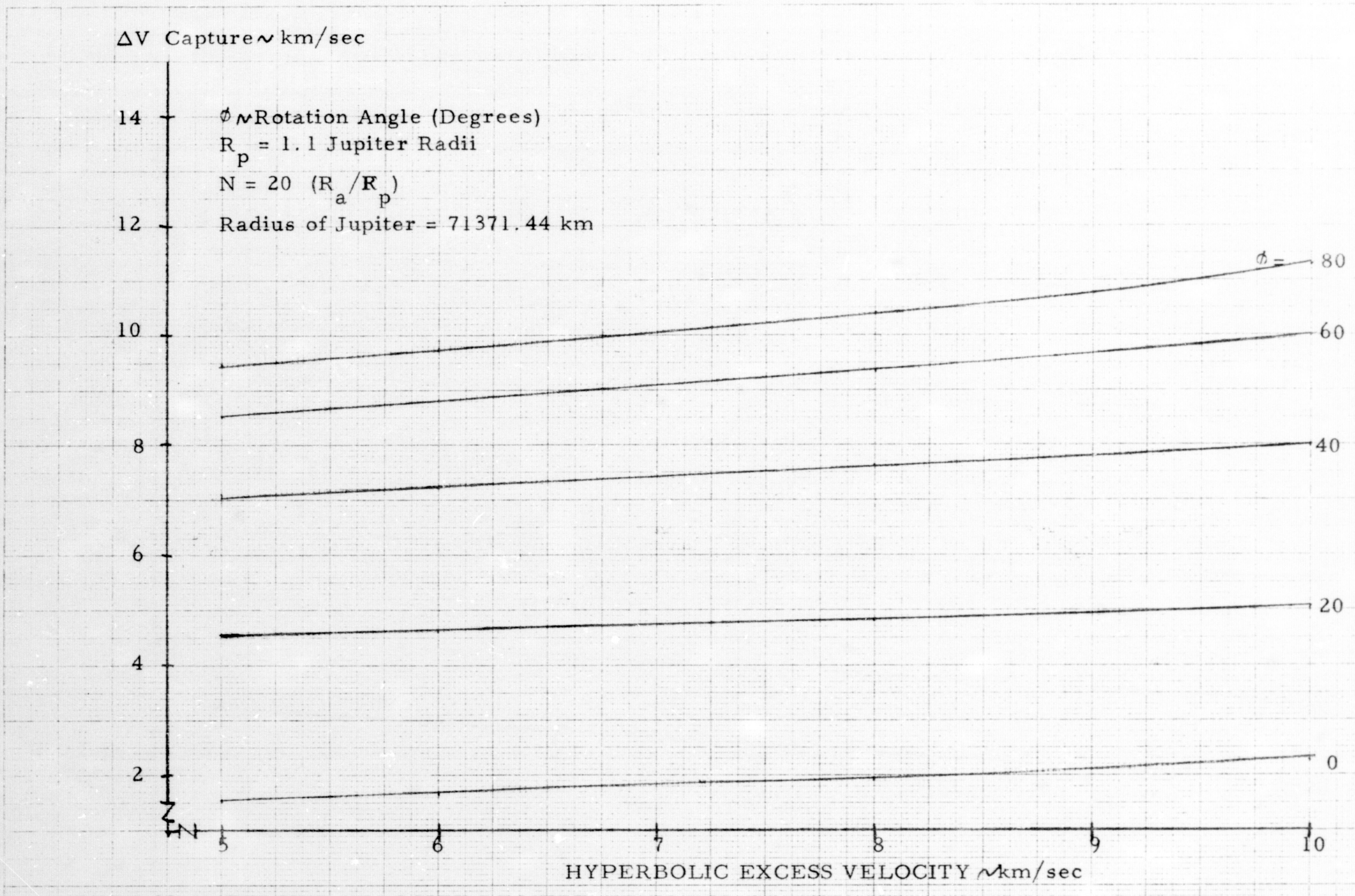


FIGURE 91 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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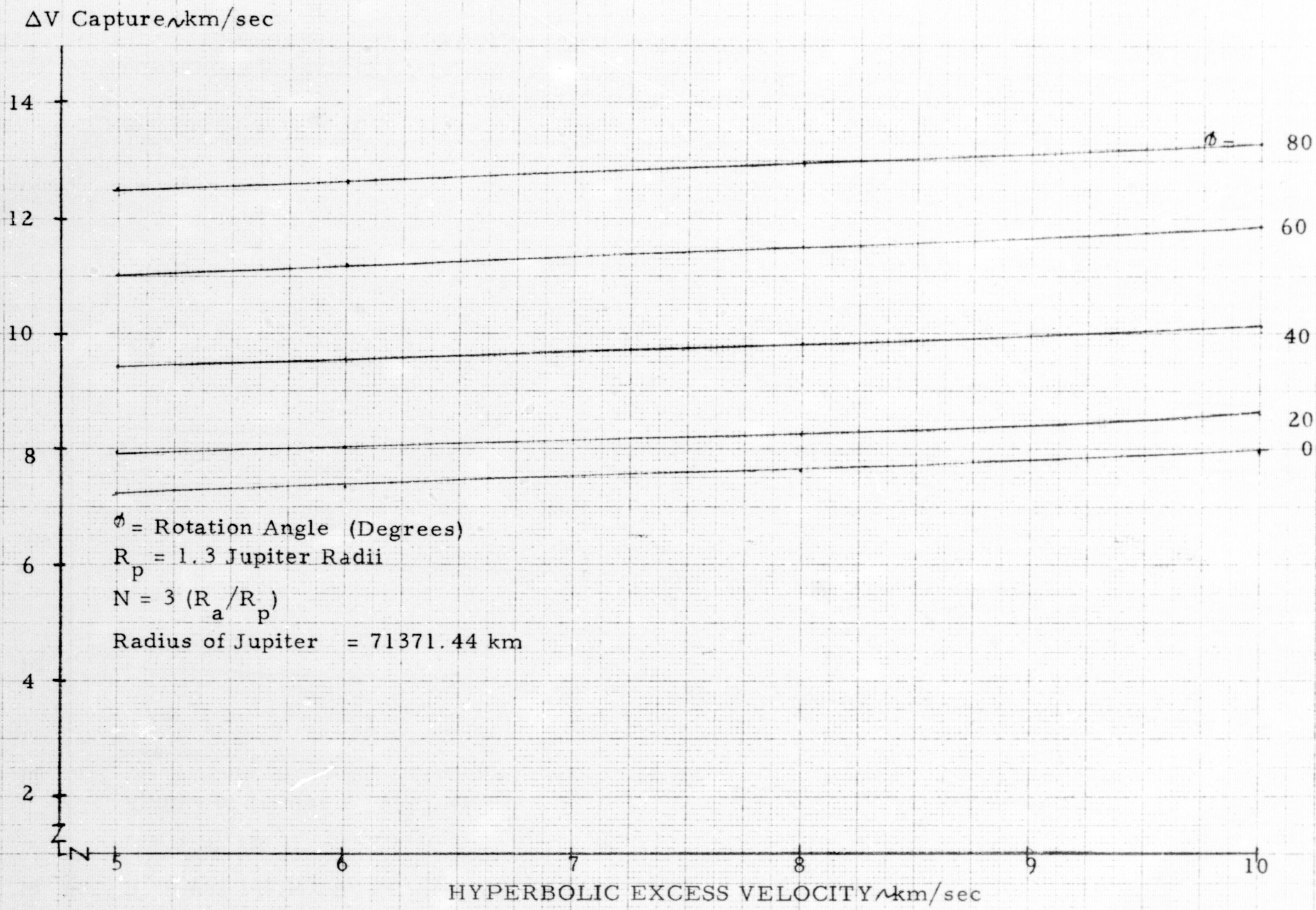


FIGURE 93 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

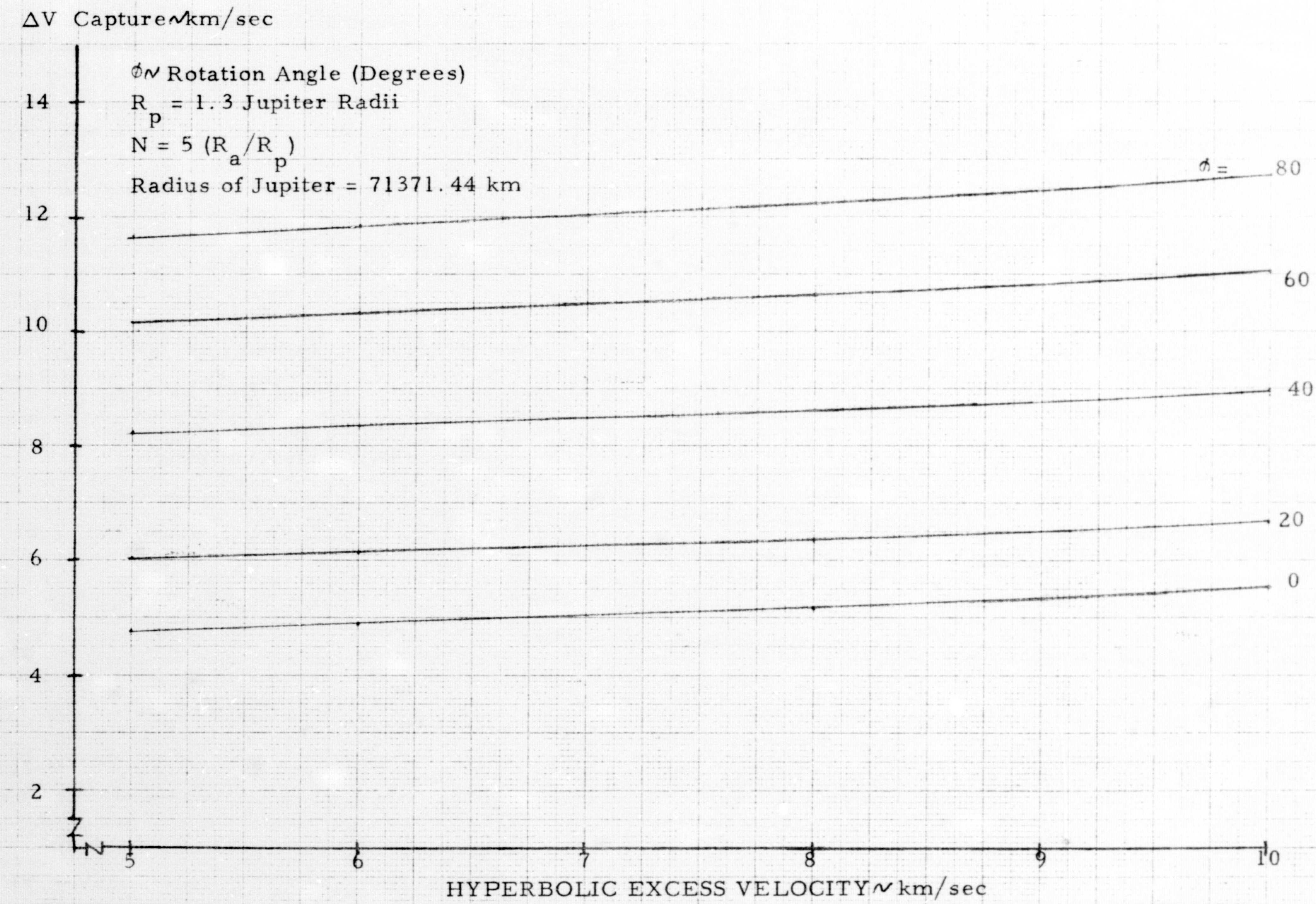


FIGURE 94 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

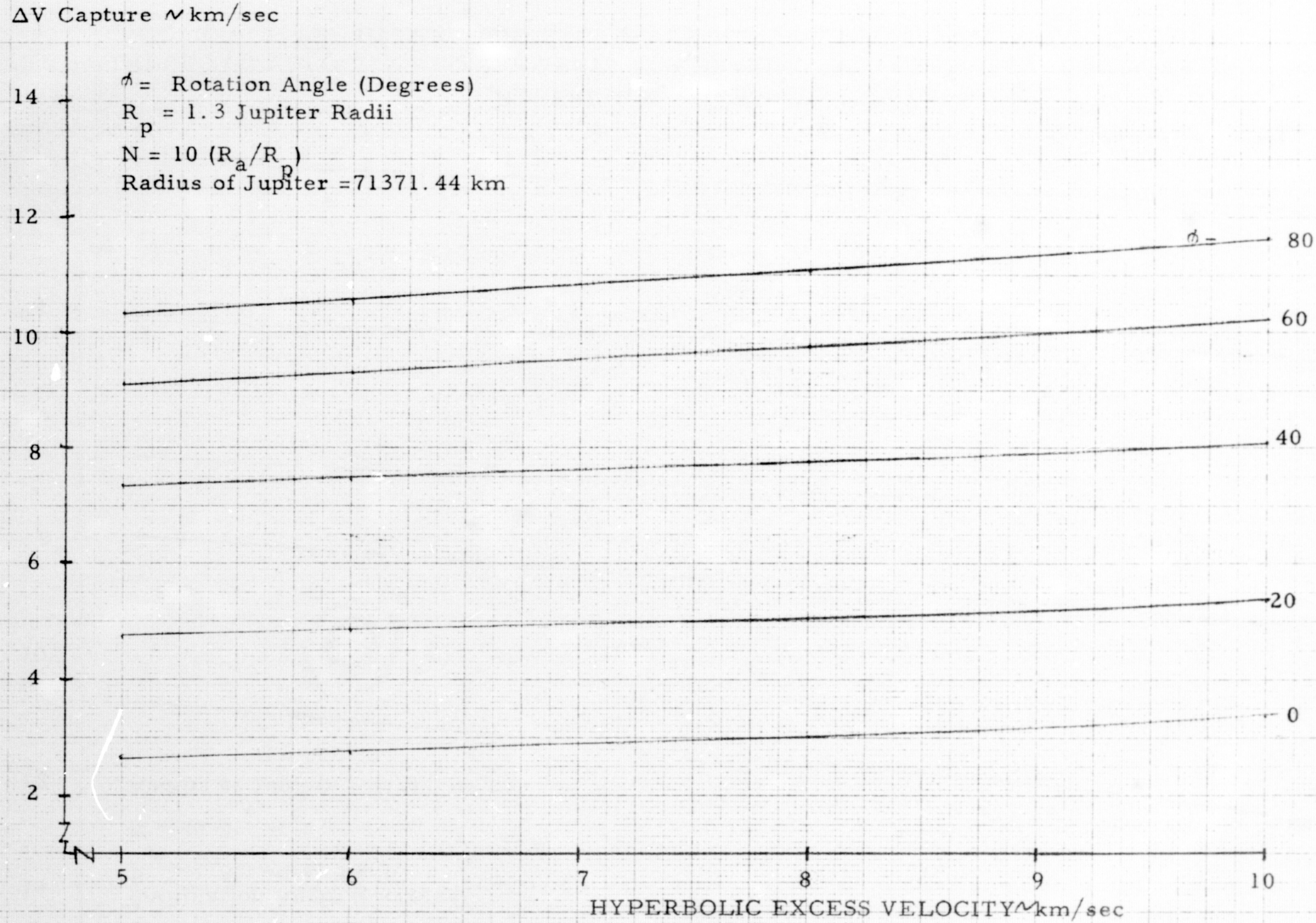


FIGURE 95 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

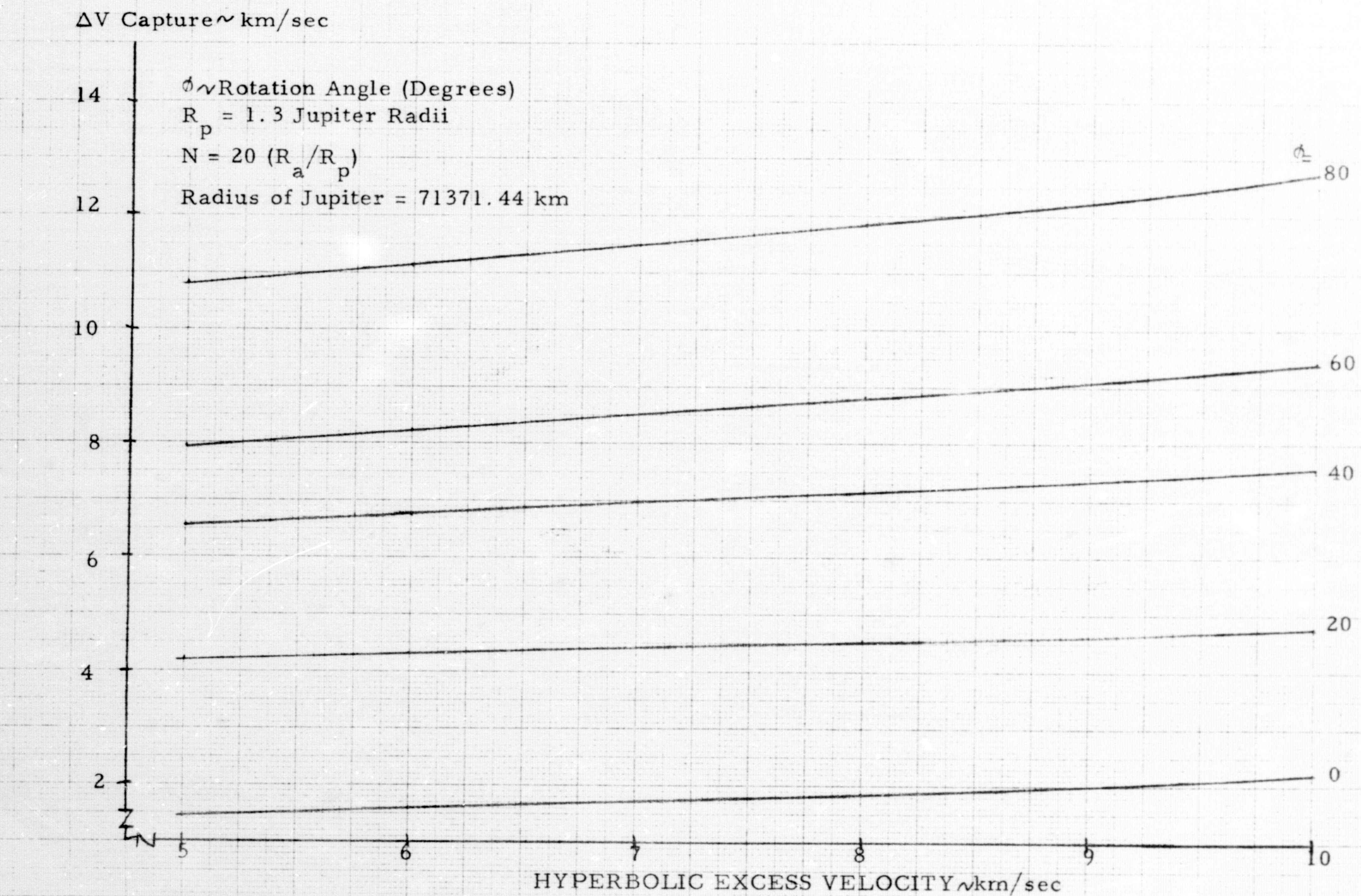
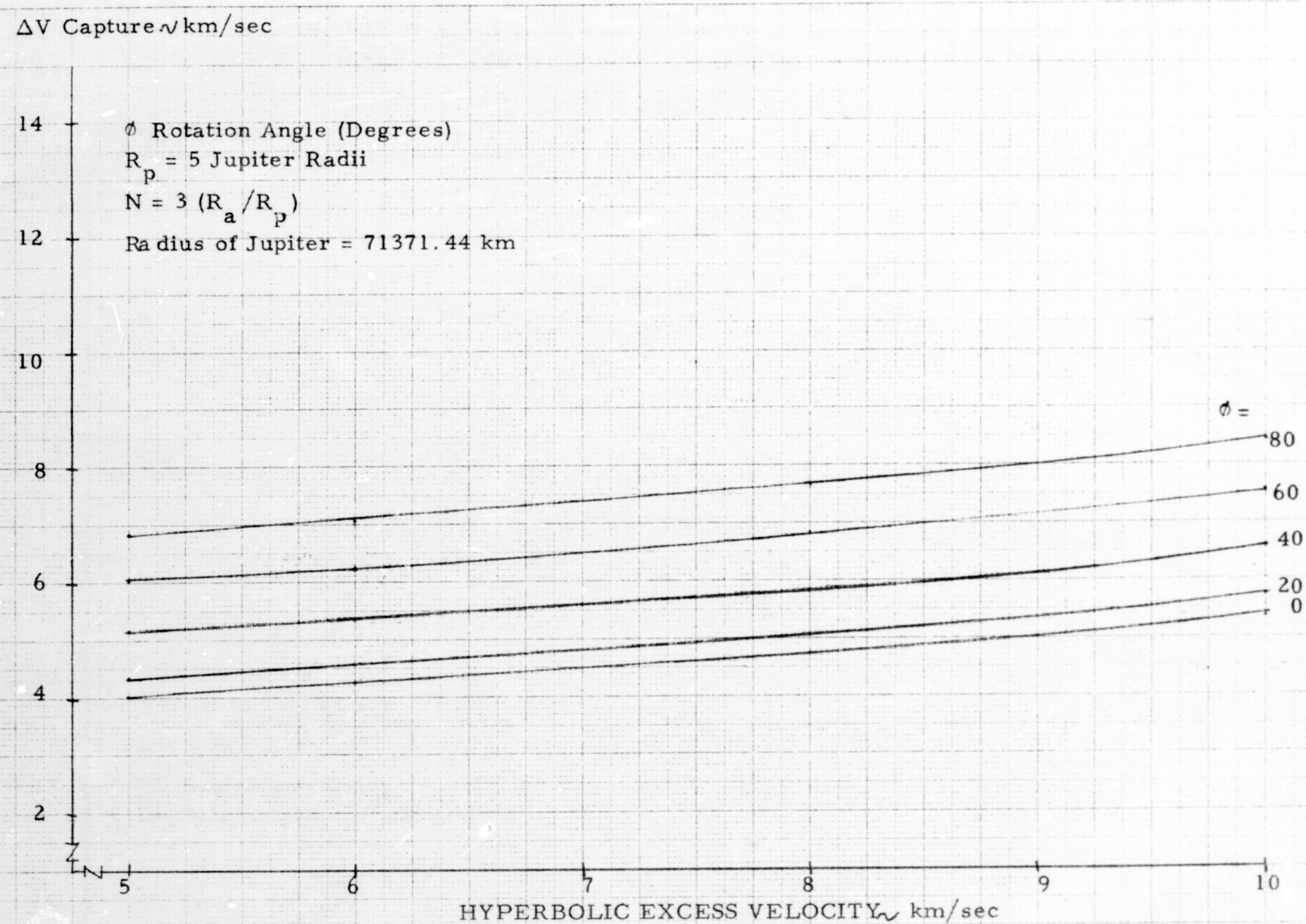


FIGURE 96 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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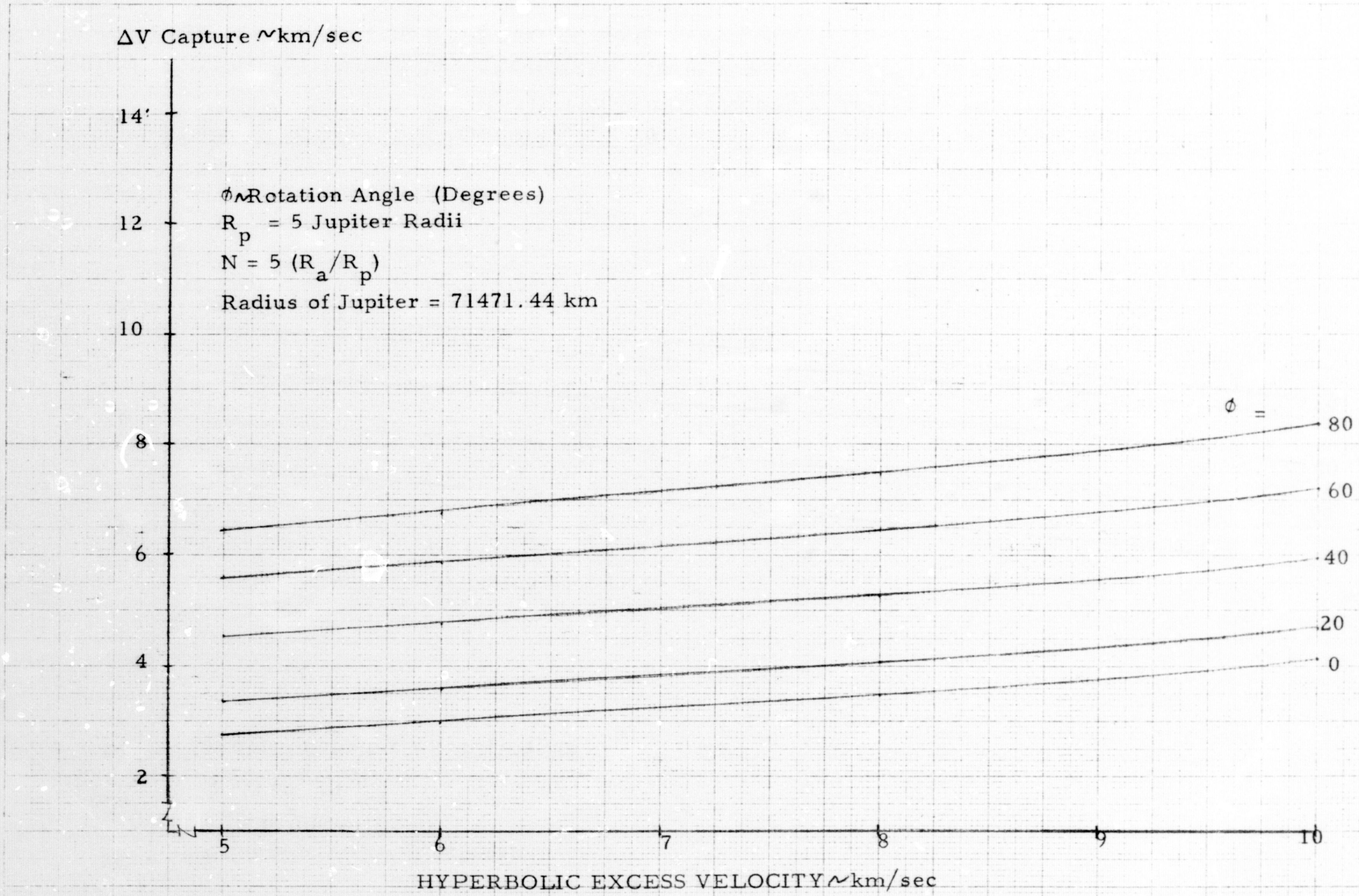
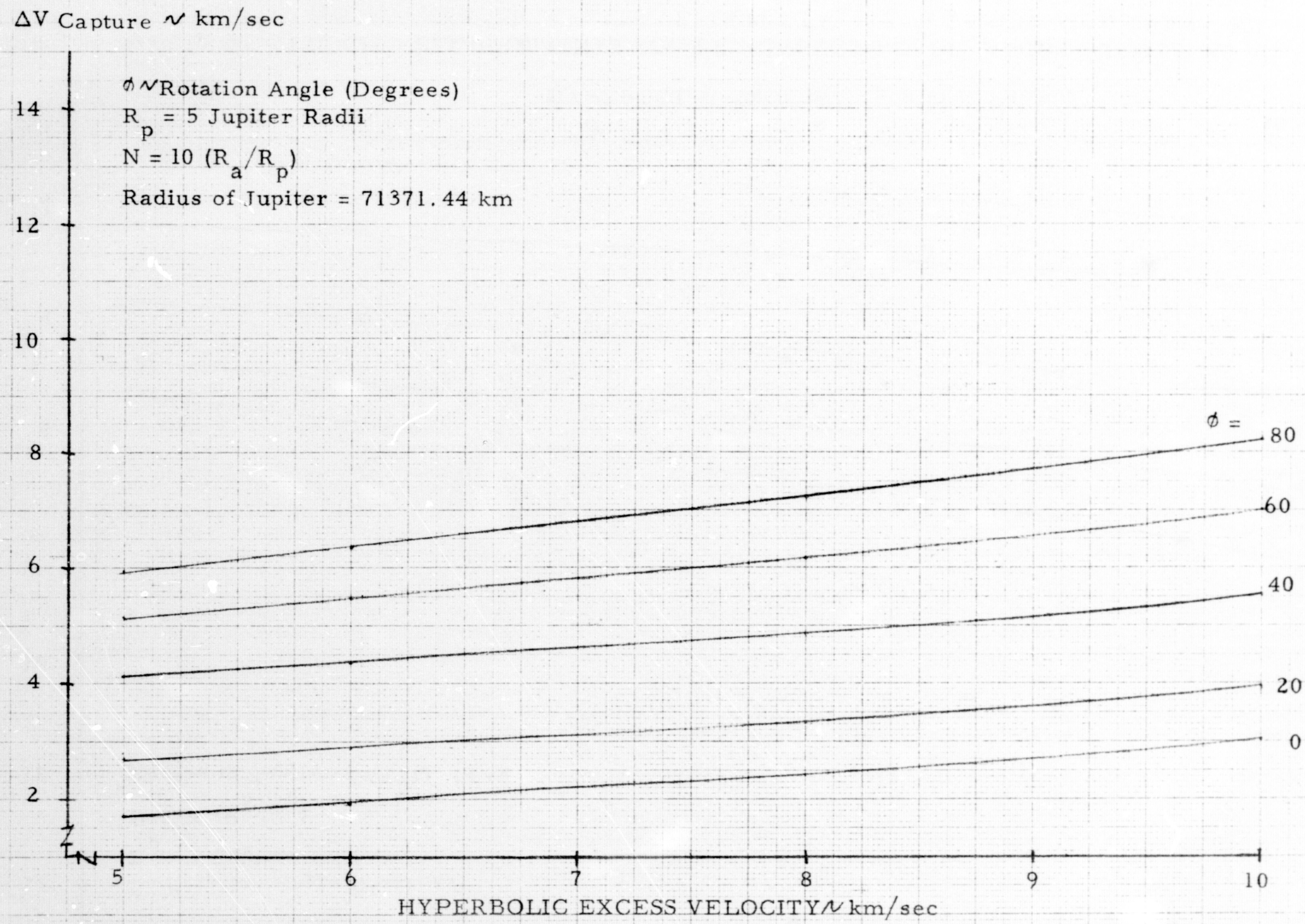


FIGURE 98 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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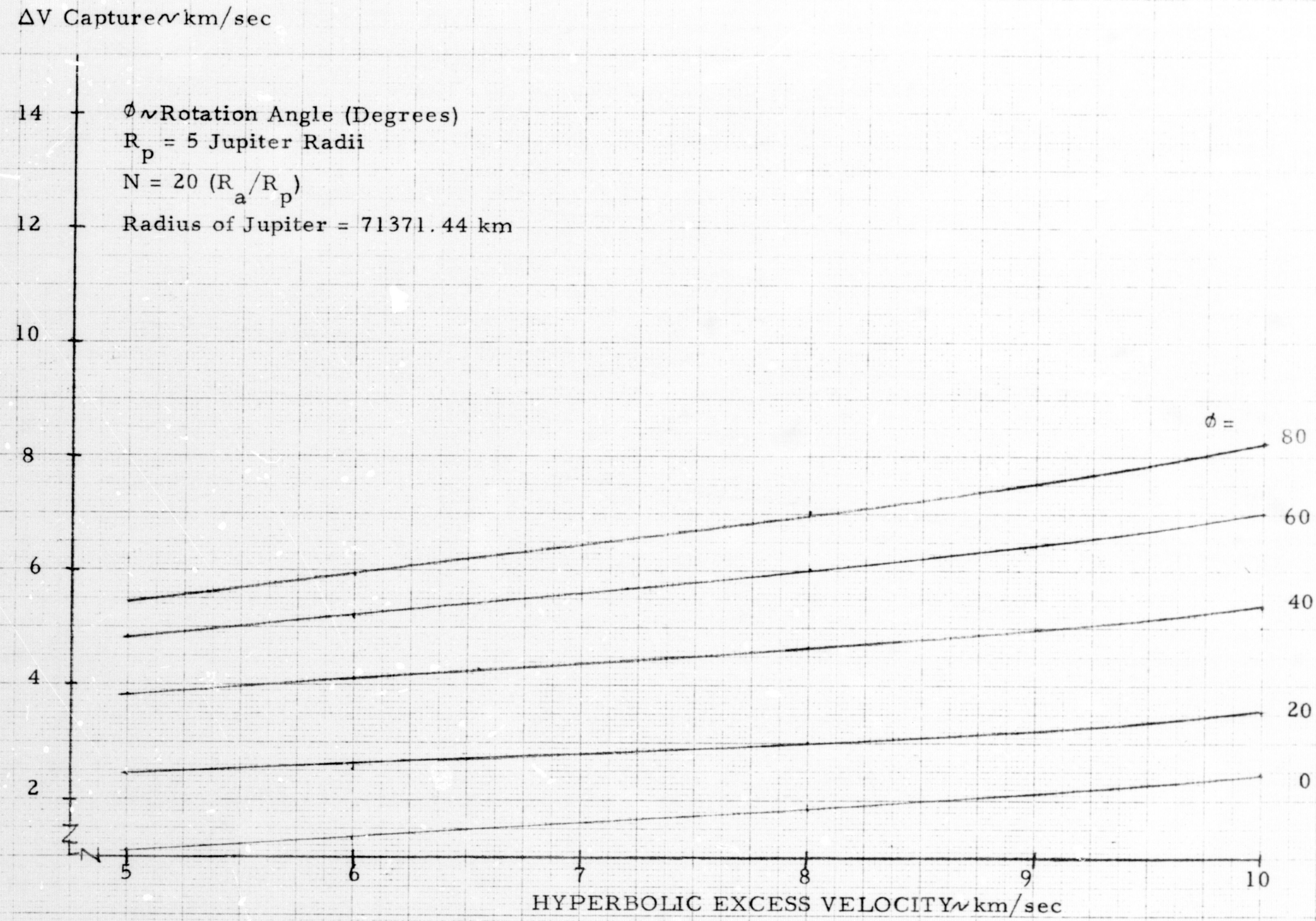
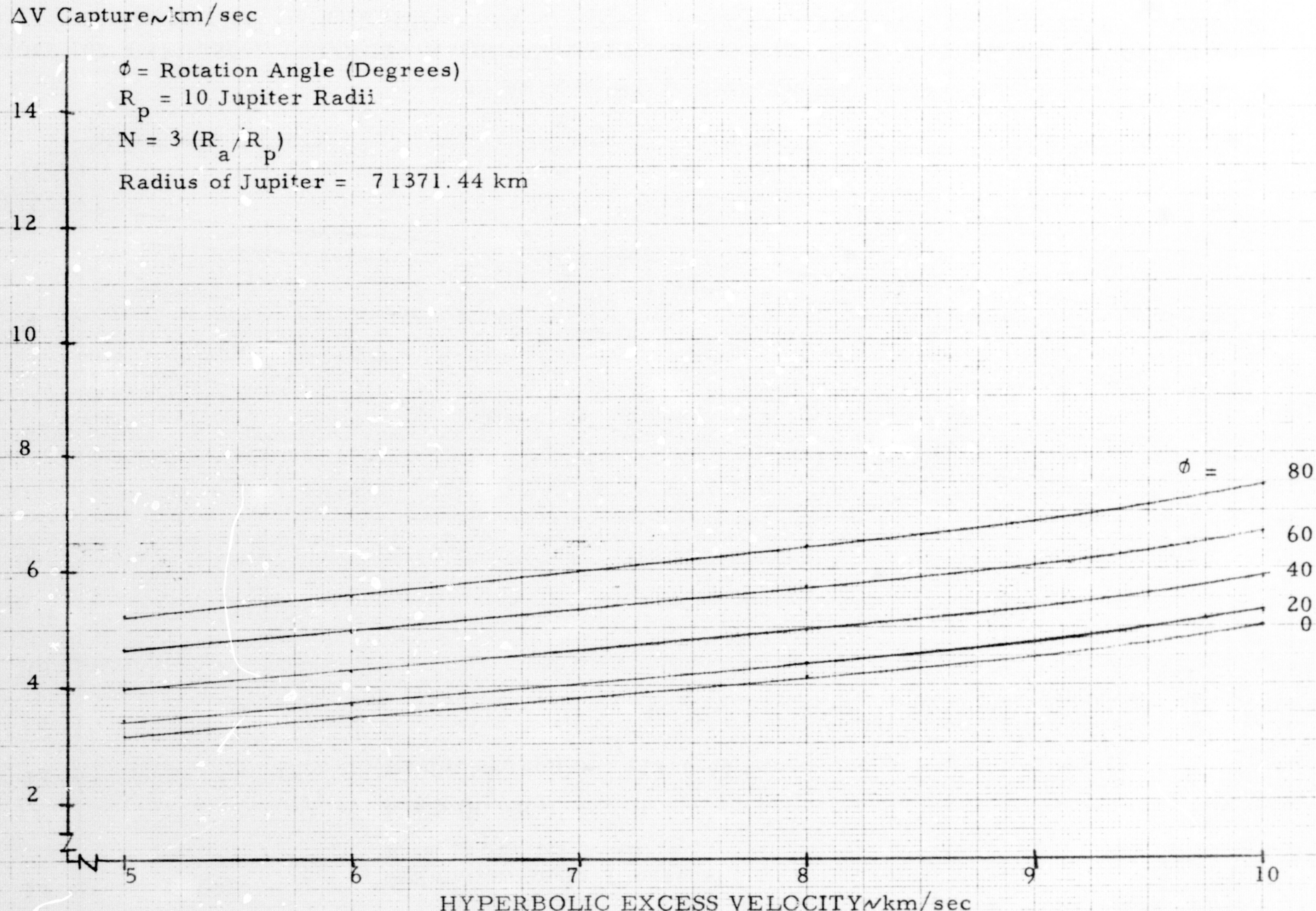


FIGURE 100 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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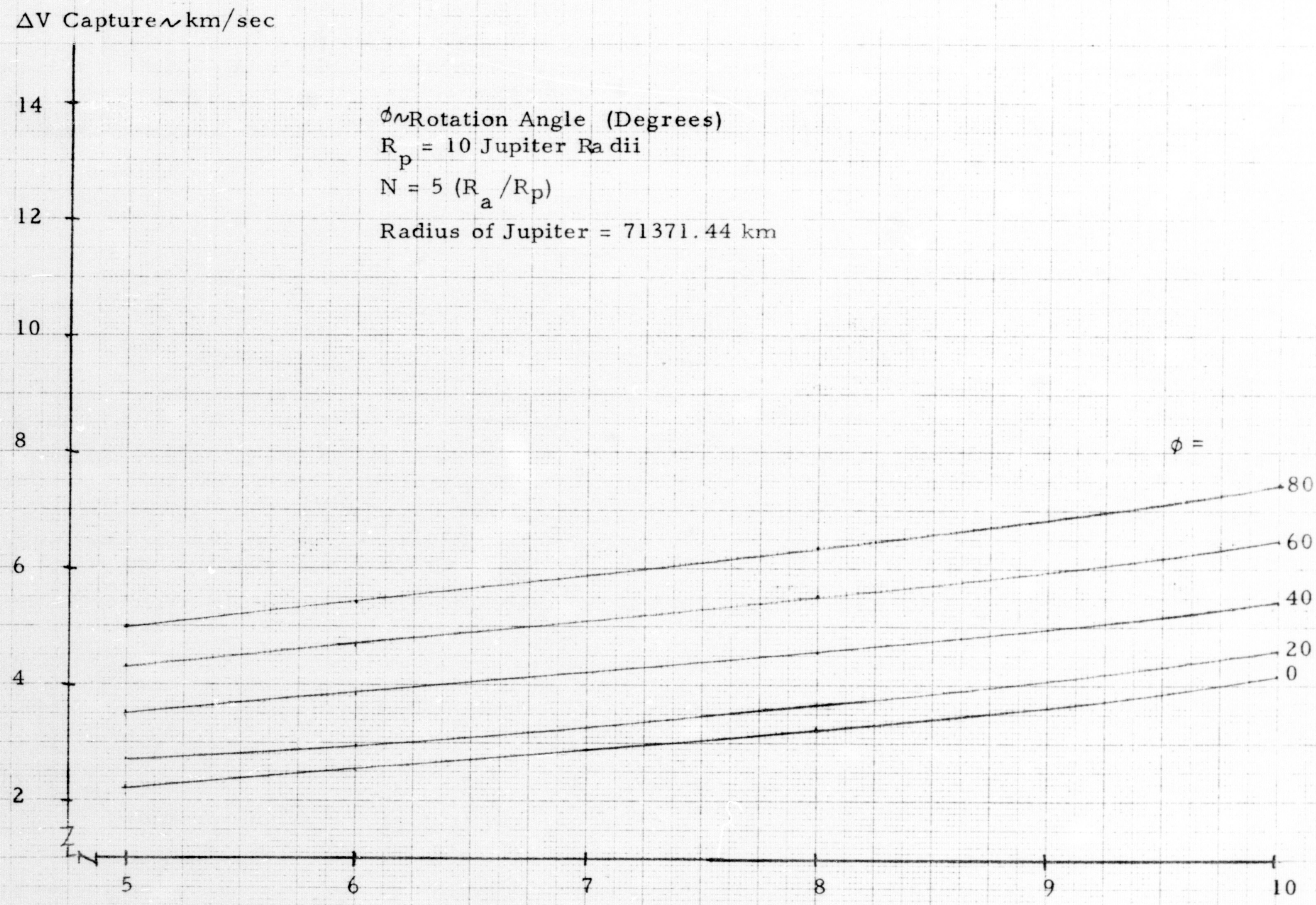
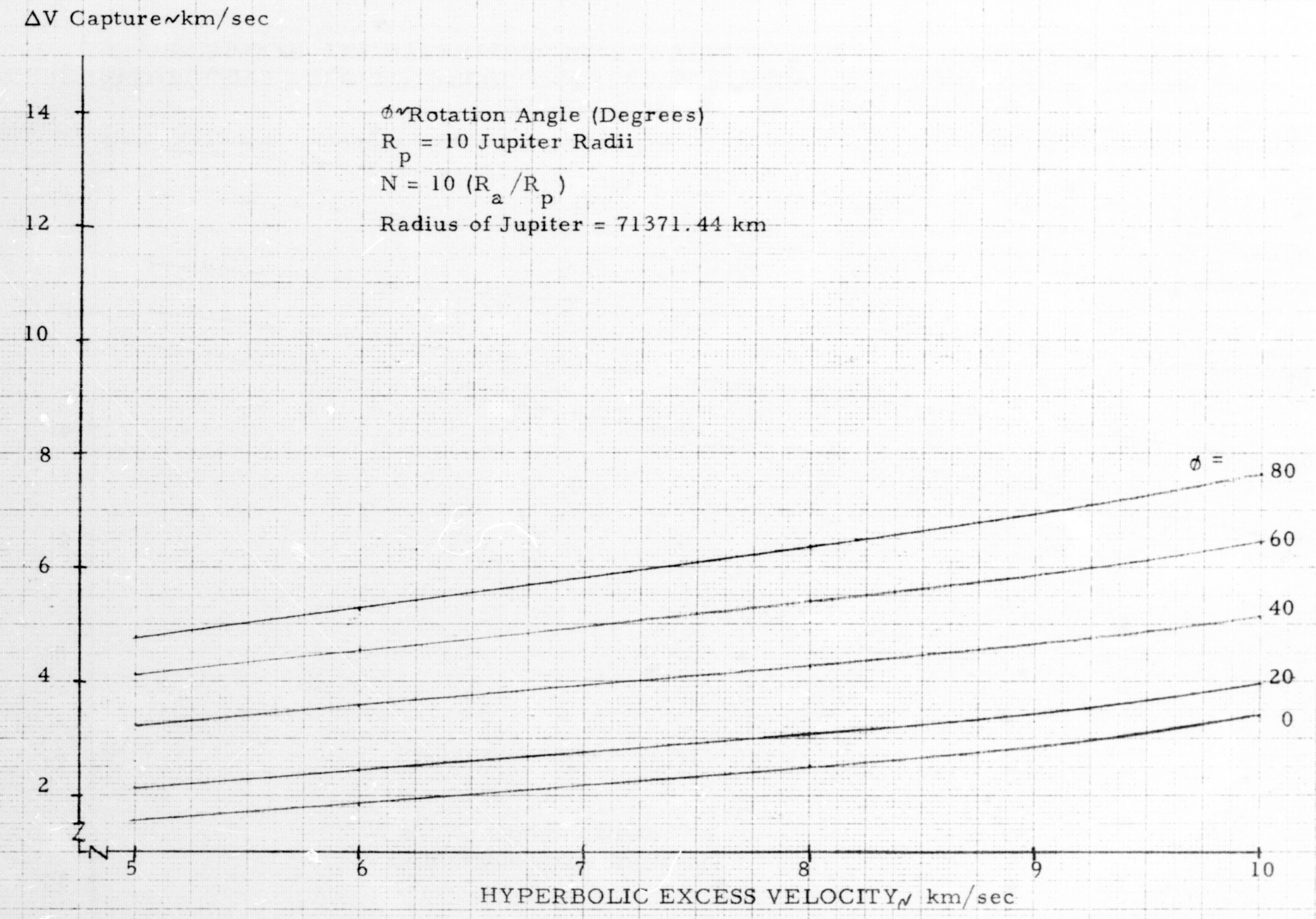


FIGURE 102 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER



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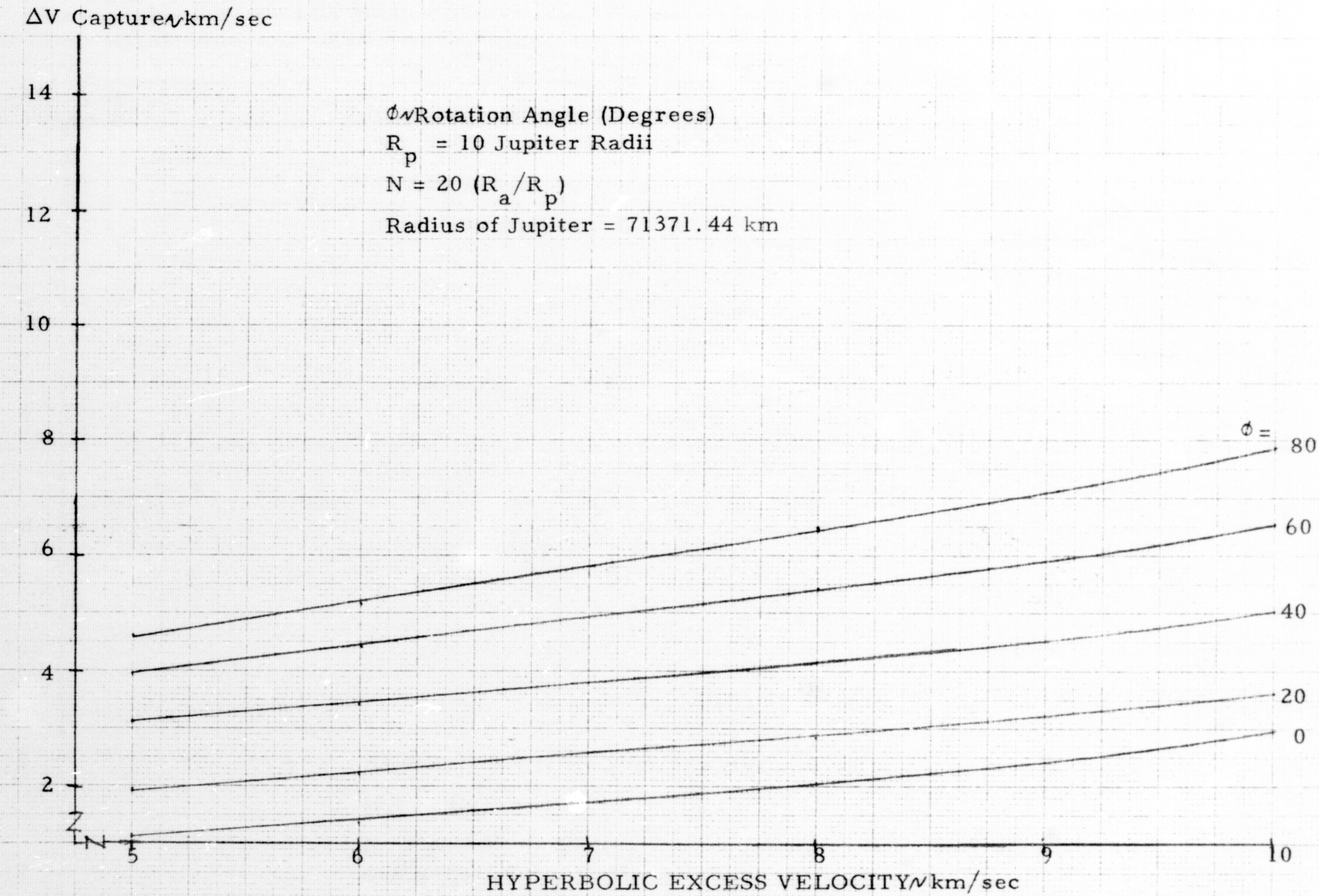

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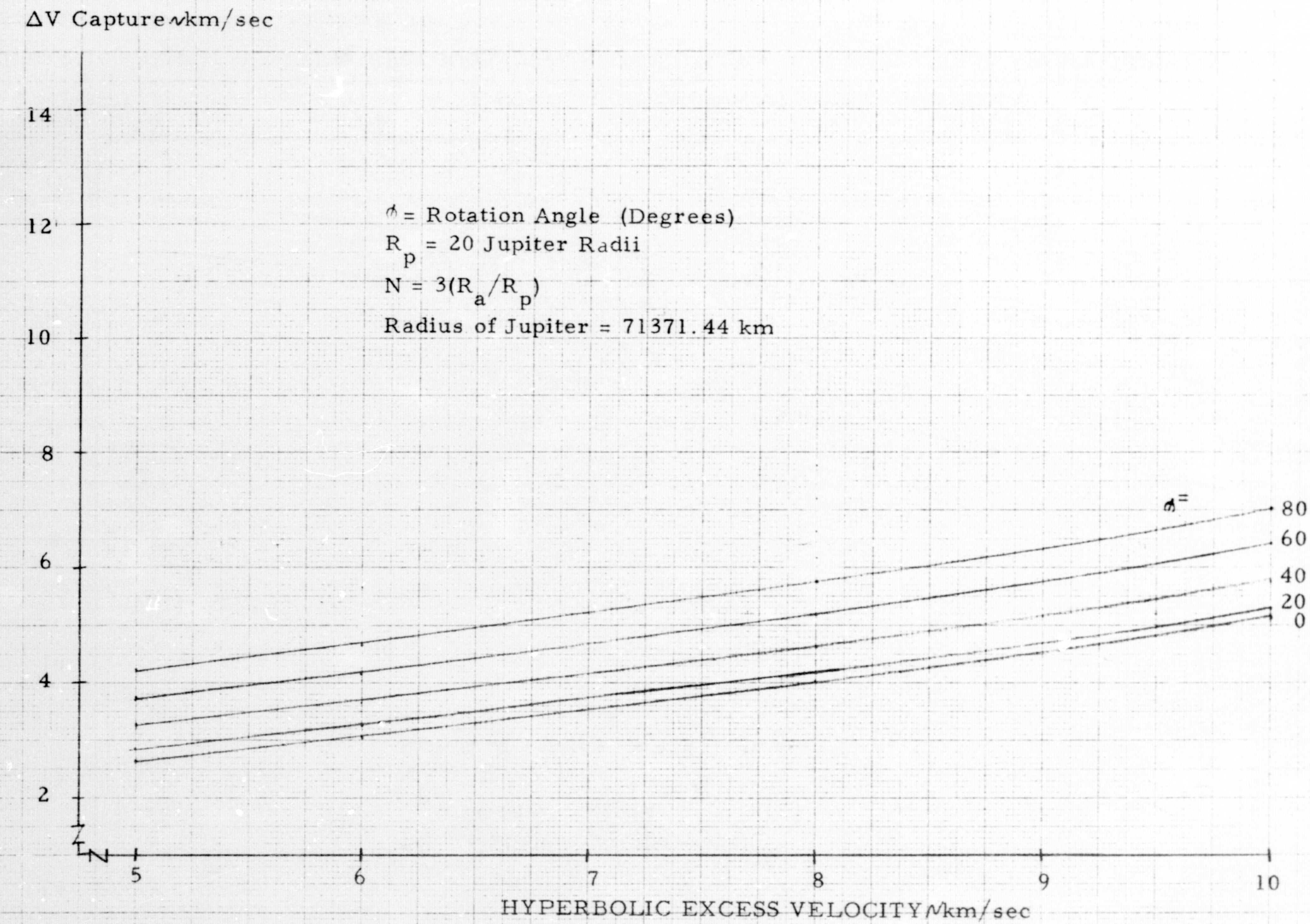

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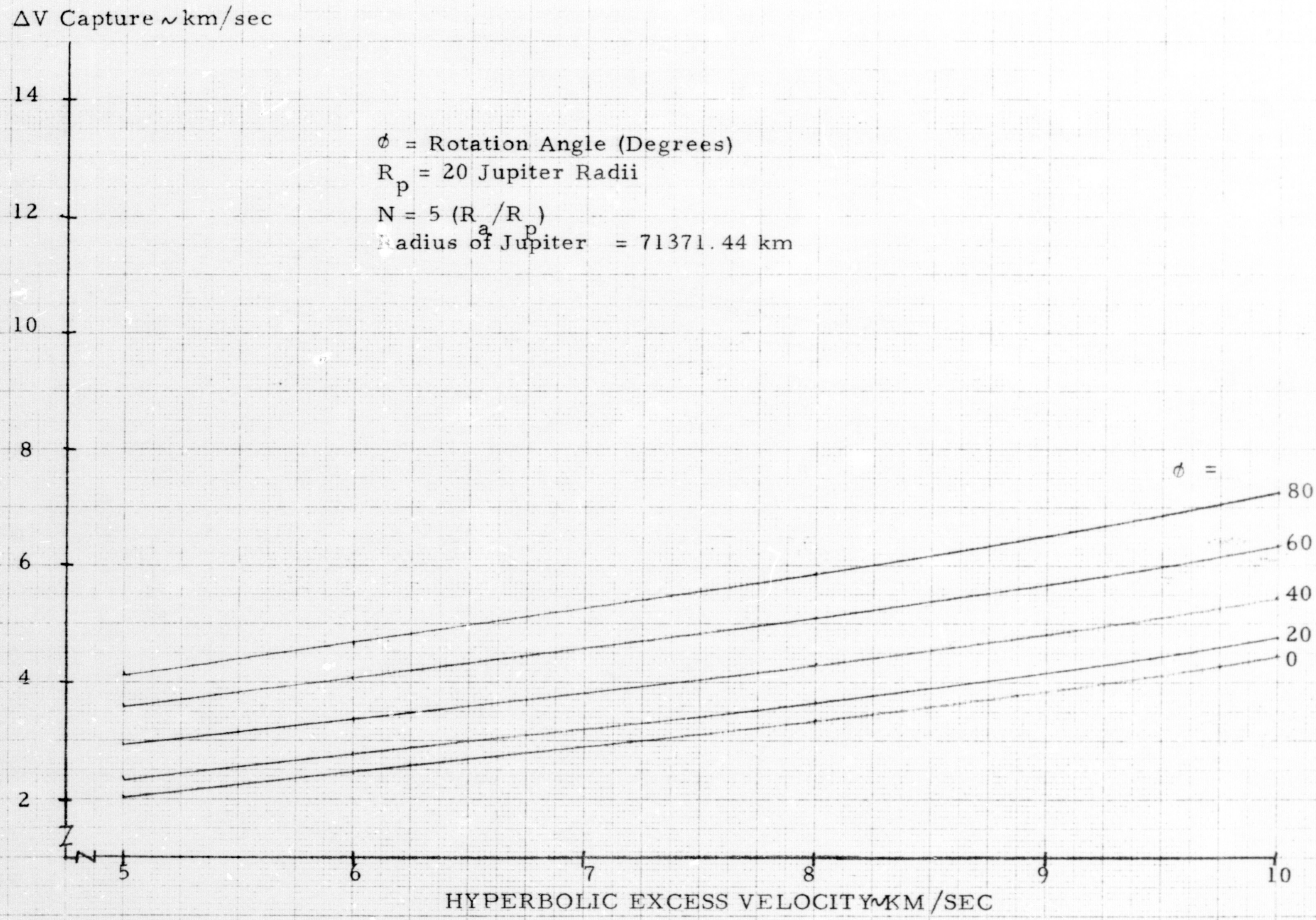

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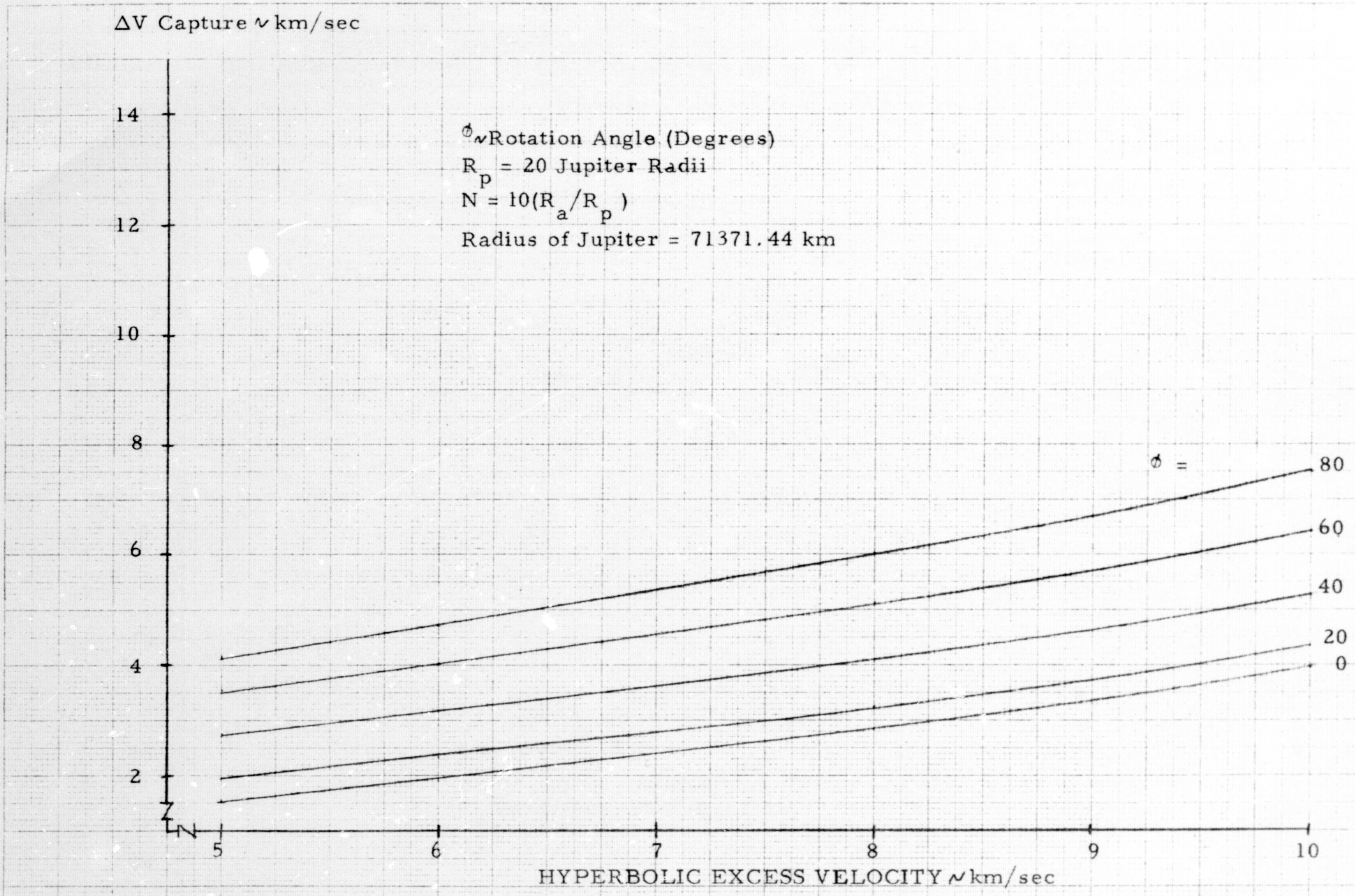

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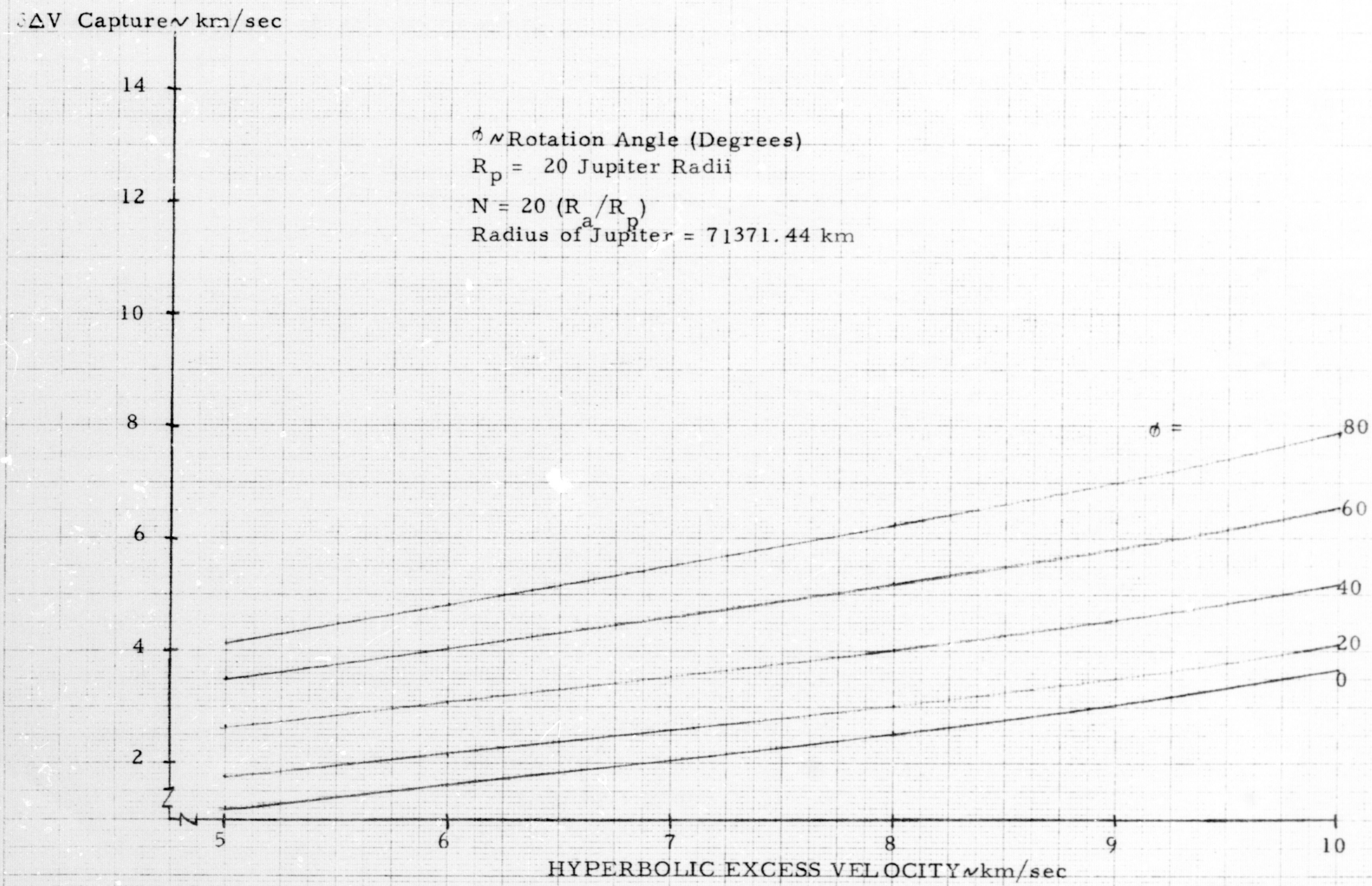

FIGURE 108 IMPULSIVE VELOCITY REQUIREMENTS FOR BRAKING INTO ORBIT ABOUT JUPITER AS A FUNCTION OF THE HYPERBOLIC EXCESS VELOCITY (V_∞) FOR TANGENTIAL TRANSFER

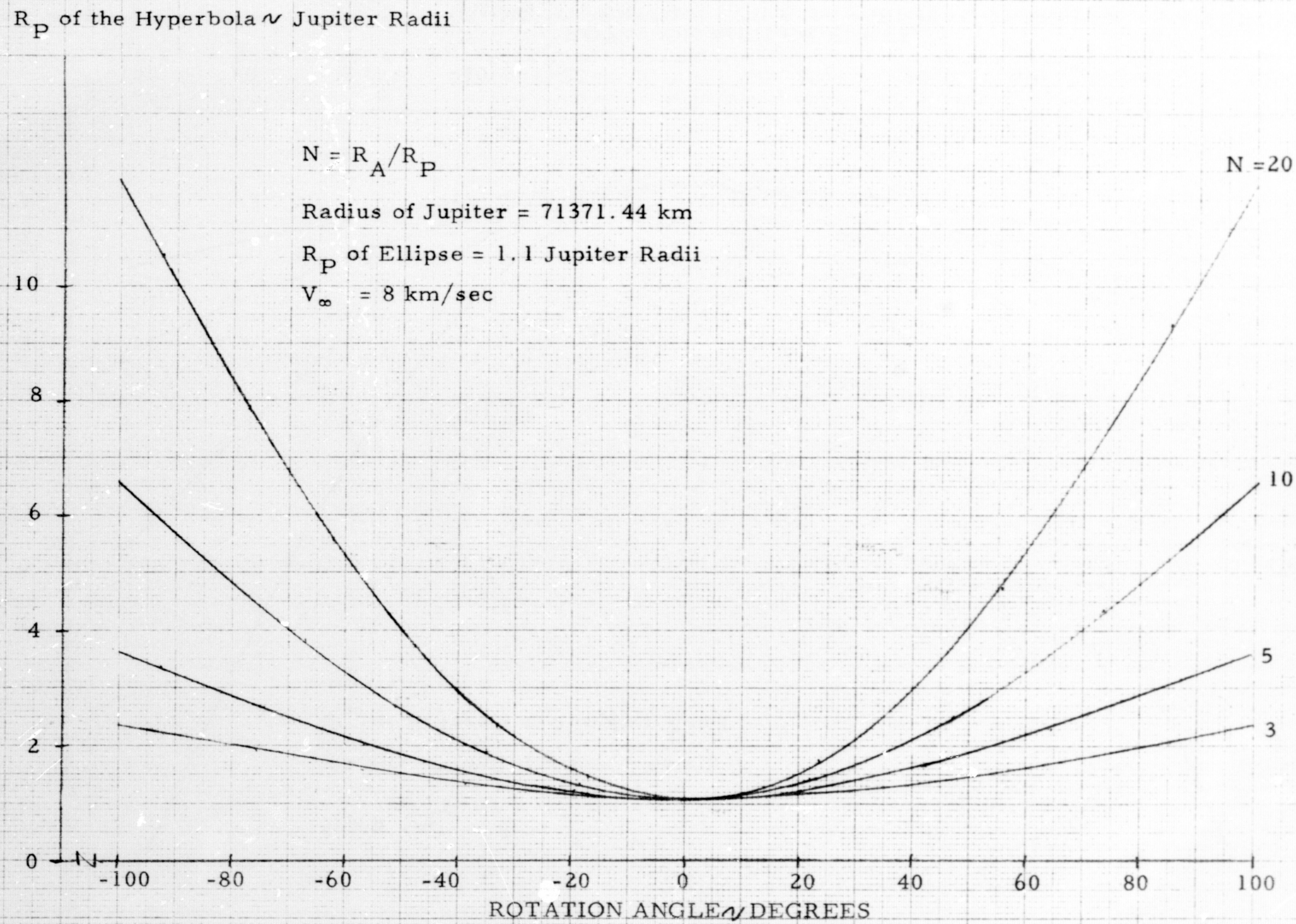

FIGURE 109 PERIAPSIS OF THE INCOMMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO JUPITER ORBIT

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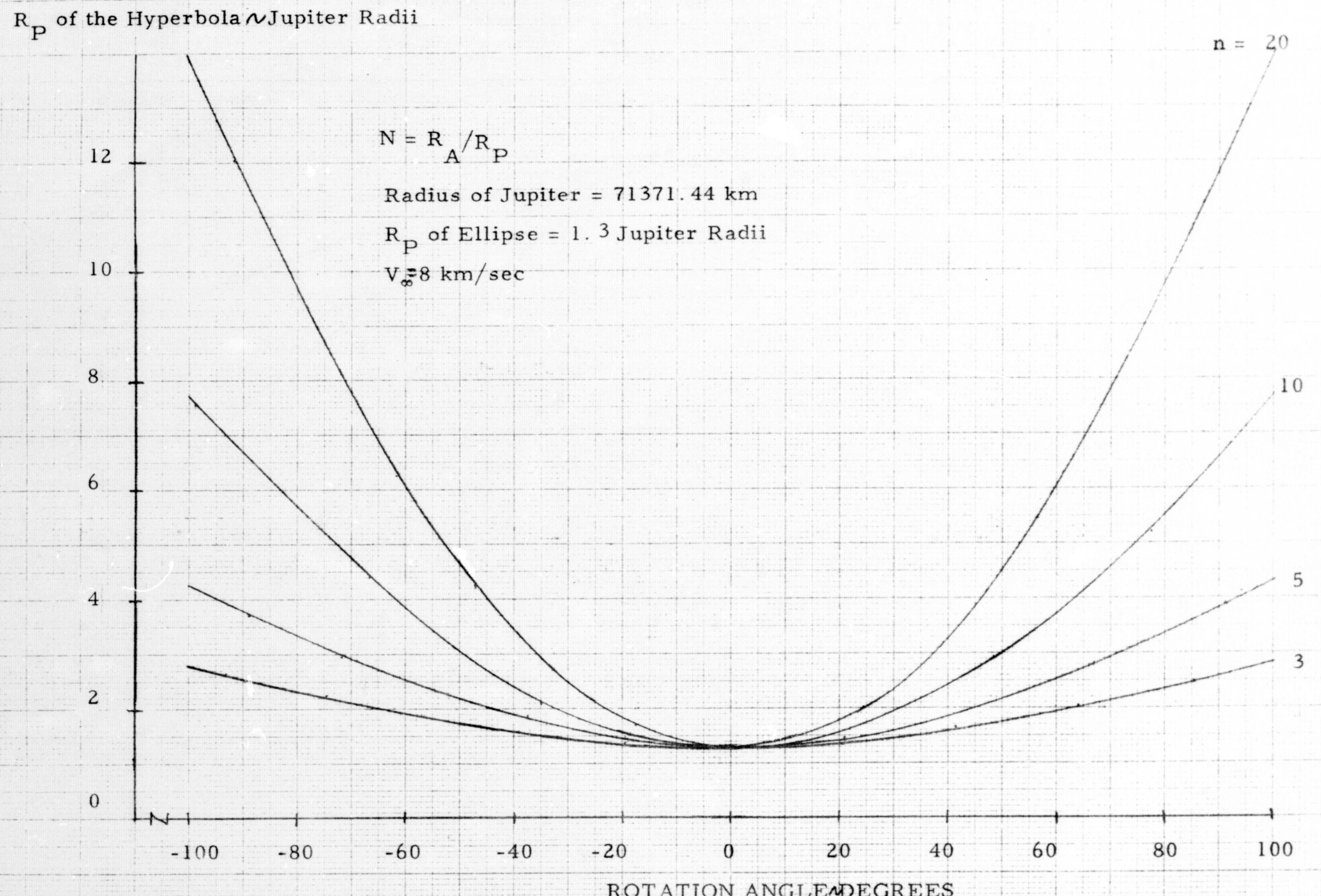

FIGURE 110 PERIAPSIS OF THE INCOMMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO JUPITER ORBIT

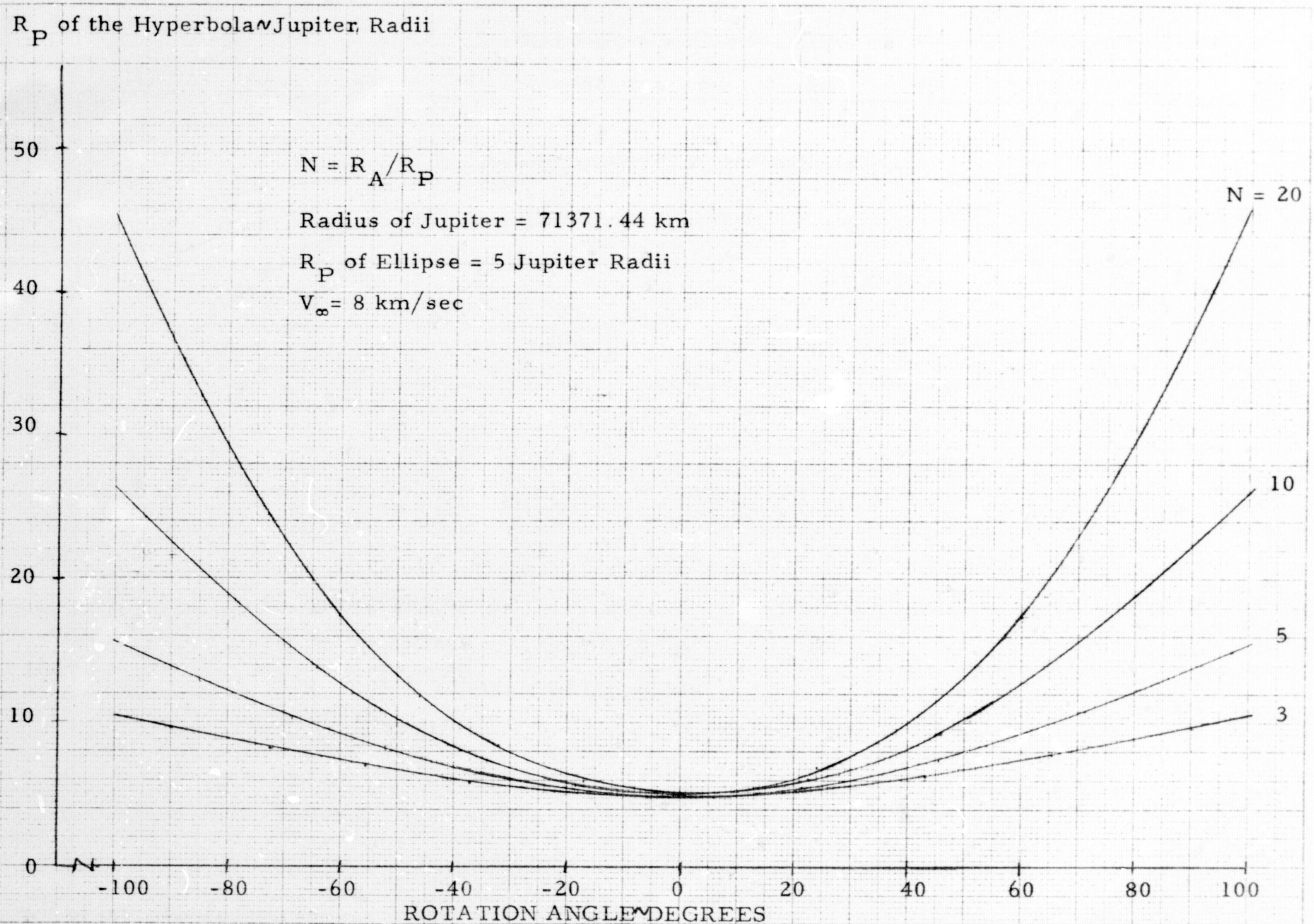

FIGURE 111 PERIAPSIS OF THE INCOMMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO JUPITER ORBIT

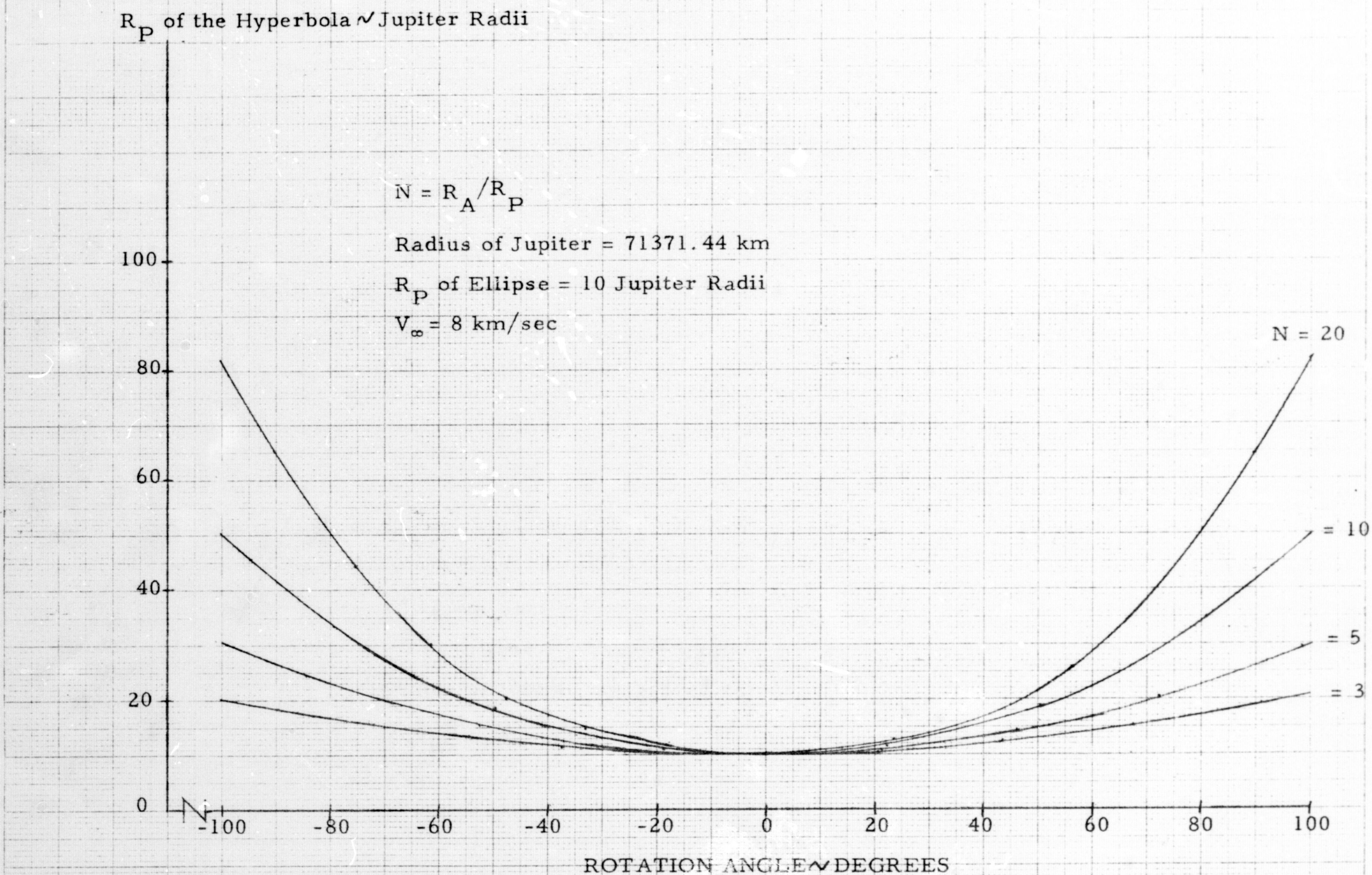

FIGURE 112 PERIAPSIS OF THE INCOMMING HYPERBOLA FOR A TANGENTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO JUPITER ORBIT

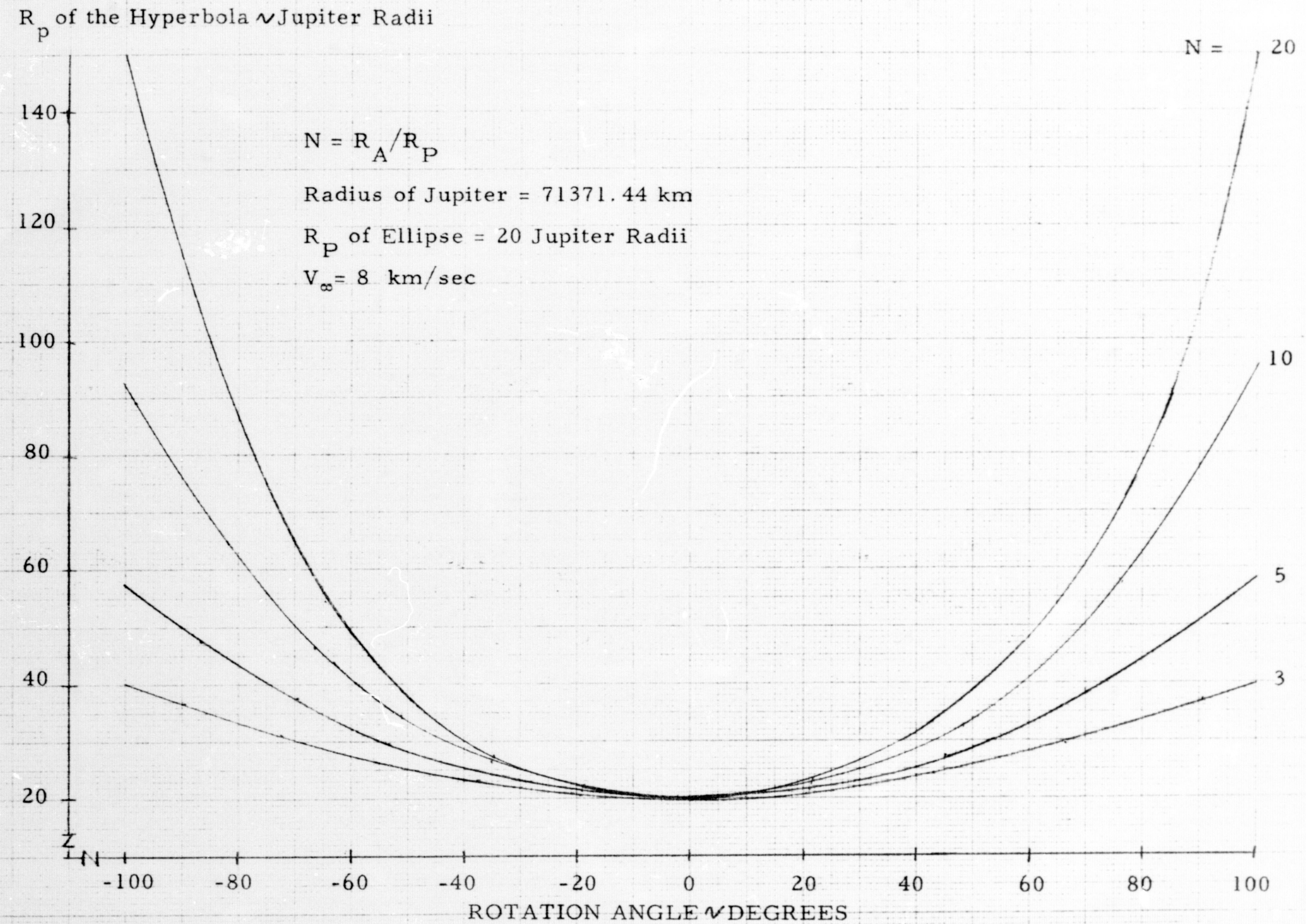

FIGURE 113 PERIAPSIS OF THE INCOMMING HYPERBOLA FOR A TANEGNTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO JUPITER ORBIT

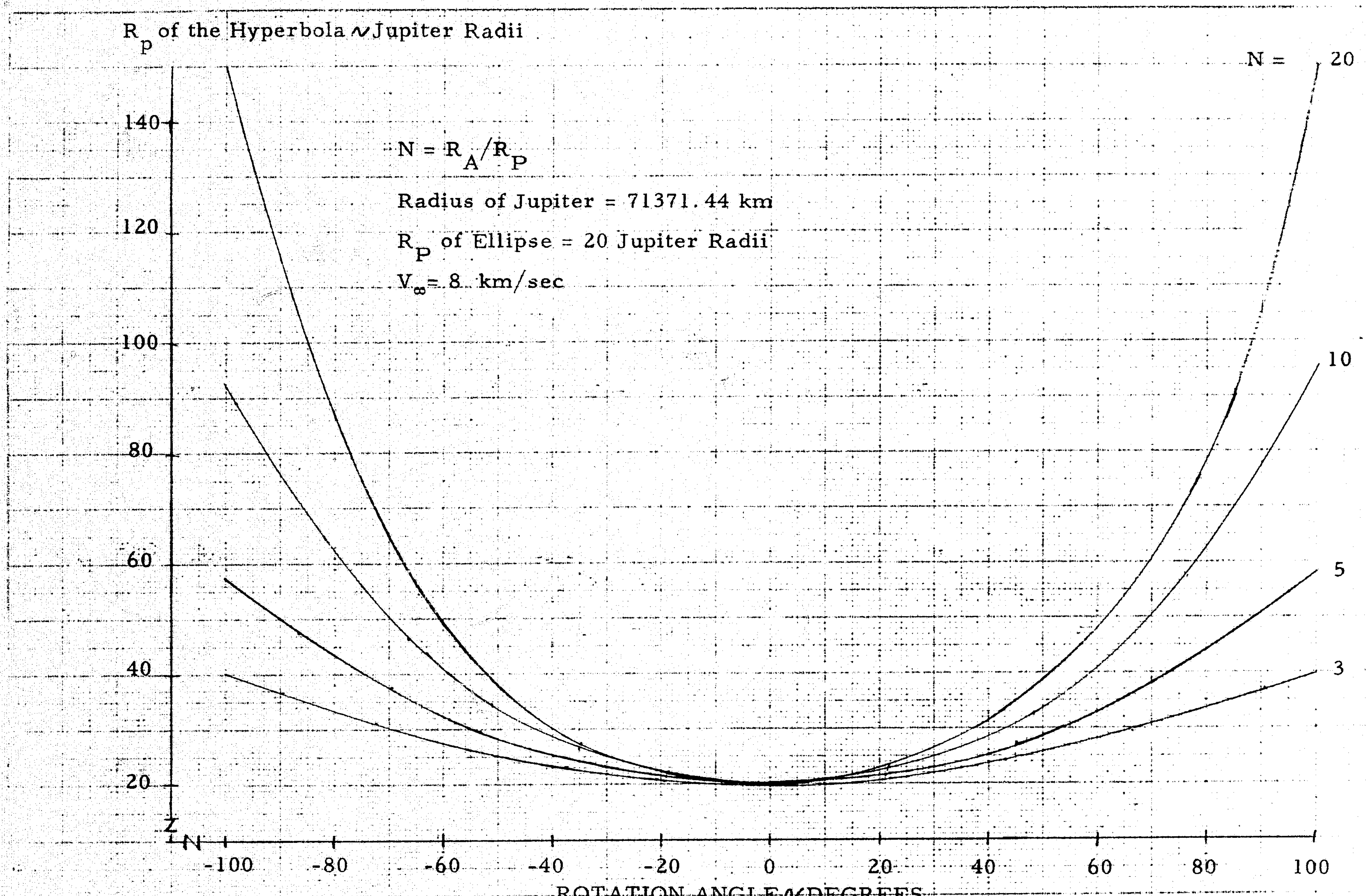

FIGURE 113 PERIAPSIS OF THE INCOMMING HYPERBOLA FOR A TANEGNTIAL TRANSFER AS A FUNCTION OF ROTATION ANGLE AND APSIDAL RATIO FOR BRAKING INTO JUPITER ORBIT

REFERENCES

1. Tito, D. A., "Analysis of In-Plane Impulsive Orbit Insertion Maneuvers," JPL, TM 312-794, March 22, 1967.

2. Wood, G. A., "Insertion Velocities Required to Enter Various Orbits About the Planet Mars," R-AERO-IN-6-67.

APPENDIX

Development of Tangential Transfer Equations

DEFINITION OF SYMBOLS

a_e	semi-major axis of the capture orbit
a_n	semi-major axis of the approach hyperbola
e_e	eccentricity of the capture orbit
e_h	eccentricity of the approach hyperbola
θ_e	true anomaly in the capture orbit
θ_h	true anomaly on the approach hyperbola
ϕ	true anomaly of the outgoing asymptote of the approach hyperbola
γ_e	flight path angle in the capture orbit measured from the local horizontal
γ_h	flight path angle on the approach hyperbola measured from the local horizontal
$(V_e)_r$	radial component of the velocity in the capture orbit
$(V_e)_t$	tangential component of the velocity in the capture orbit
$(V_h)_r$	radial component of the velocity on the incoming hyperbola
$(V_h)_t$	tangential component of the velocity on the incoming hyperbola
ΔV	magnitude of the impulsive velocity required for orbit insertion
Γ	the direction of the velocity increment measured from the local horizontal

Impulsive insertion into a capture orbit can occur only when the incoming hyperbola and the capture orbit intersect. Near-minimum velocity requirements occur when the two orbits intersect tangentially. For this reason, only tangential results are presented.

The analysis presented here is given in JPL's TM 312-794, dated March 22, 1967. The equations are presented here for reference.

The orbit transfer occurs when the radius of the orbits intersects (i.e., when re = rh) or when

$$\frac{a_e(1 - e_e^2)}{1 + e_e \cos \theta_e} = \frac{a_h(e_h^2 - 1)}{1 + e_h \cos \theta_h}, \tag{1}$$

where

$$a_e = \frac{r_p(1 + N)}{2} \tag{2}$$

$$e_e = \frac{N - 1}{N + 1} \tag{3}$$

$$a_h = \frac{\mu}{V\infty} \tag{4}$$

$$e_h = 1 + \frac{r_{ph}}{a_h} \tag{5}$$

$$\theta_e = \theta_h + \pi - \psi - \cos^{-1}(1/e), \tag{6}$$

and the rotation angle, ROT,

$$ROT = \theta_h - \theta_e = \psi + \emptyset - \pi \tag{7}$$

where

$$\emptyset = \cos^{-1}(1/e_h) \tag{8}$$

is the true anomaly of the incoming asymptote.

Substituting equation (6) into (1) yields:

$$\frac{a_e(1 - e_e^2)}{1 + e_e \cos[\theta_h + \pi - \psi - \emptyset]} = \frac{a_h[e_h^2 - 1]}{1 + e_h \cos \theta_h} \tag{9a}$$

or

$$\frac{a_e(1 - e_e^2)}{1 + e_e[\cos \theta_h \cos A - \sin \theta_h \sin A]} = \frac{a_h[e_h^2 - 1]}{1 + e_h \cos \theta_h}, \tag{9b}$$

where

$$A = \pi - \psi - \emptyset = -ROT \tag{10}$$

and

$$\cos A = \frac{1}{e_h} [\sqrt{e_h^2 - 1} \sin \psi - \cos \psi]. \tag{11}$$

By multiplying through by the denominators in equation (9b) and collecting terms as a function of first $\sin \theta_h$ and $\cos \theta_h$, we obtain

$$\sin \theta_h = \frac{-DB \pm C\sqrt{-B^2 + C^2 + D^2}}{C^2 + D^2} \tag{12a}$$

$$\cos \theta_h = \frac{-BC \mp D\sqrt{-B^2 + C^2 + D^2}}{C^2 + D^2} \tag{12b}$$

where

$$B = a_e(1 - e_e^2) - a_h(e_h^2 - 1) \tag{13a}$$

$$C = e_h\ a_e(1 - e_e^2) - e_e\ a_h(e_h^2 - 1) \cos A \tag{13b}$$

$$D = e_e\ a_h(e_h^2 - 1) \sin A. \tag{13c}$$

In obtaining solutions for θ_h from equations (12a) and (12b), the (+) sign on the square root in (12a) must go with the (-) sign on the radical in equation (12b) in order to satisfy the identity:

$$\sin^2\theta_h + \cos^2\theta_h = 1.$$

Equations (12a) and (12b) can yield two, one, or no solutions, depending upon whether $(-B^2 + C^2 + D^2)$ is positive, zero, or negative. If it is negative, there is, of course, no solutions. If it is zero, only the tangential solution exists.

When $(-B^2 + C^2 + D^2)$ is positive, two solutions exist, given by the two signs in (12a) and (12b). Each of these solutions are used to calculate the following:

$$\theta_e = \theta_h + A. \tag{14}$$

The distance from the center of the planet is

$$r = \frac{a_e(1 - e_e^2)}{1 + e_e \cos \theta_e} \tag{15a}$$

$$r = \frac{a_h(e_h^2 - 1)}{1 + e_h \cos \theta_h}. \tag{15b}$$

The velocities are

$$V_e = \sqrt{\mu/r\ (2 - r/a_e)} \tag{16a}$$

$$V_h = \sqrt{2\mu/R + V\infty^2}\ . \tag{16b}$$

The flight path angles measured from the horizontal at the point of transfer for the ellipse and the hyperbola are given by the equations:

$$\gamma_e = \text{arc}\ \tan[e_e \sin \theta_e/(1 + e_e \cos \theta_e)] \tag{17a}$$

$$\gamma_h = \text{arc}\ \tan[e_h \sin \theta_h/(1 + e_h \cos \theta_h)]. \tag{17b}$$

The radial and transverse components of the velocity impulse are given by:

$$(V_e)r = V_e \sin \gamma_e \tag{18a}$$

$$(V_e)t = V_e \cos \gamma_e \tag{18b}$$

$$(V_h)r = V_h \sin \gamma_h \tag{18c}$$

$$(V_h)t = V_h \cos \gamma_h. \tag{18d}$$

The magnitude of the insertion velocity increment (ΔV) and the angle between ΔV and the local horizontal (Γ) is given by

$$\Delta V = \sqrt{((V_{hr}) - (V_e)_r)^2 + ((V_h)_t - (V_e)_t)^2} \tag{19}$$

and

$$\Gamma = \tan^{-1}\left[[(V_e)_r - (V_h)_r]/[(V_e)_t - (V_h)_t]\right]. \tag{20}$$

If $-B^2 + C^2 + D^2 = 0$, there is only one transfer possible, i.e., the tangential. Substituting (13a), (13b), and (13c) into the equation and letting

$$E = \frac{a_e}{a_h^2} [a_e(1 - e_e^2) + 2a_h + 2e_e a_h \cos \psi + 2e_e^2 a_e \sin^2\psi] \tag{21}$$

and

$$F = \frac{d_e}{d_h^2} [a_o(1 - e_e^2) + 2a_h + 2e_e a_h \cos \psi], \tag{22}$$

we obtain

$$e_h = \sqrt{E \pm \sqrt{E^2 - F^2} + 1}. \tag{23}$$

Only one solution is valid and is checked by solving the two equations for the radius. The invalid solution yields two different values for the radius, and thus is discarded.